A TREATISE

OF THE

PROPERTIES OF ARCHES,

AND THEIR

ABUTMENT PIERS:

CONTAINING

Propositions for describing geometrically the Catenaria, and the Extradosses of all Curves, so that their several Parts and their Piers may equilibrate;

ALSO

CONCERNING BRIDGES, AND THE FLYING BUTTRESSES OF CATHEDRALS.

To which are added, in Illustration,

SECTIONS OF TRINITY CHURCH, ELY; KING'S COLLEGE CHAPEL, CAMBRIDGE; WESTMINSTER ABBEY; SALISBURY, ELY, LINCOLN, YORK, AND PETERBOROUGH CATHEDRALS.

BY SAMUEL WARE, ARCHITECT.

"True, indeed, the great load above of the walls, and vaulting of the "nave, should seem to confine the pillars in their perpendicular station, "that there should be no need of butment inward; but experience hath "shewn the contrary."—Sir C. Wren's Survey of Salisbury Cathedral.

"Et è converso solæ catenariæ sunt fornices sive arcus legitimi: et "cujuscunque alterius figuræ arcus ideo sustinetur, quod in illius cras-"sitie quædam catenaria inclusa sit."—Phil. Trans. 1697. Dr. David Gregory.

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1809.

PREFACE.

THE inquiry into the theory of the equilibration of arches has hitherto been almoſt excluſively confined to mathematicians; and the moſt celebrated of the preſent age have thought it worthy their attention.

The difference of opinion which exiſts on this ſubject may be attributed to the circumſtance, that practical men have not joined in the inveſtigation.

Much inconvenience has been experienced by the architect and engineer from ſome inconſiſtencies in the treatiſes which have been publiſhed on arches and piers, and alſo from the tedious calculations neceſſary to apply the principles contained in them to the caſes of conic ſections and mixed curves.

This inveſtigation was commenced for the author's own information, and for a particular purpoſe; but he has been led to purſue the ſubject further than he intended, and now offers the following eſſay to the public, with a hope that ſome of the defects in it will be aſcribed to the interruptions which other purſuits have neceſſarily occaſioned.

In the early progreſs of this treatiſe, the paper of Dr. David Gregory on the catenaria, Phil. Tranſ. 1697, induced a conviction, that if a ſimple mode of deſcribing that line geometrically could be diſcovered, it would be an important ſtep towards the attainment of a correct theory of the equilibration of arches, and their abutment piers. This difficulty, after an expenſe of much time, the author has ſurmounted, (ſee Prop. ix. *fig.* 24.) and a theory anſwering his expectation has been thence deduced.

After this work had been written, it was conceived, that an apt reference to exiſting buildings

buildings would be a plain and effectual mode of eftablifhing the truth of what is there advanced: Trinity Church, Ely, and King's College Chapel, Cambridge, were, therefore, fixed upon for this purpofe; and an opportunity was taken, which profeffional bufinefs in the neighbourhood afforded, to make the drawings *figures* 26. and 27. The fection in Price's Obfervations on Salifbury Cathedral led to the propofition on flying buttreffes.

All the fections herein publifhed (with the exception of that of Salifbury cathedral) have been made on purpofe for this tract, from accurate admeafurements; and much time and expenfe have been beftowed in obtaining them. The author thinks that, in prefenting them to the public, he fhall afford to the antiquary fubjects for curious fpeculation, and to the architect a ftudy from which he may derive fome practical knowledge.

Thefe drawings evidently fhew that the catenaria has been ufed to afcertain the thicknefs of the columns and abutments in thefe edifices.

Under the head of Introductory Definitions and Remarks, the word equipollence is adopted as a general term, applicable to all arches which are enabled to retain their actual ftate; whether from the equilibration of their parts, or from the mode in which they are connected. As the theory of the equilibration of arches, and the properties of the catenaria, are deduced from the laws of motion, thofe laws, which are directly applicable to thefe fubjects, are given in the firft fection; it being conceived, that the convenience of a prompt reference to them, if neceffary, would facilitate to the reader his examination of the fyftem propofed to be fupported in the fubfequent fheets.

The author takes this opportunity of ftating, that from an inveftigation of domes of equilibration, and their abutment walls, he has reafon to believe, that the outline of the

fpace

ſpace encloſed by a number of chains of equal links, connected together, but free to move according to gravitation, ſuſpended from a circular hoop, bears the ſame relation to a dome and its abutment wall, as the catenaria does to an arch and its abutment piers; though he has not had leiſure to aſcertain the truth of this hypotheſis. He is not aware that the curve, which would be formed in this manner, has already been conſidered by mathematicians: ſhould, however, any thing have been publiſhed concerning it, he will be obliged to any gentleman who ſhall refer him to ſuch a work.

14, John Street, Adelphi, London,
Jan. 1, 1809.

CONTENTS.

INTRODUCTORY

DEFINITIONS AND REMARKS.

Fig. 1. AN arch of equipollence is a curve, whofe feveral parts are prevented from following their natural directions towards the centre of the earth by mutual oppofition. The refpective actions of the parts are fo regulated, that through their reciprocity they muft remain in their actual ftate; or, if they begin to gravitate at the fame time, each part muft move through the fame perpendicular fpace in the fame time, and the chord A B will always keep its parallelifm. It is, therefore, of no importance to the ftability or form of the arch of what confiftency the foundations A, B are; fo that they are equally compreffible, and defcend together, keeping their horizontal plane. Unlefs the parts A, B move, no other part can move.

If the lines of direction of all the parts centered in one point, that point muſt receive an equal preſſure from each part.

An arch may be equipollent from either of two cauſes: the one from the relation of the weights of the moveable parts, independent of any aſſiſtance from continuity, as in an arch of equilibration; the other from continuity, when the parts do not equilibrate.

Equipollence in an arch does not imply that every part is equally able to ſuſtain a given weight, but that it is able to bear an equal additional relative weight on each part, or that the continuity may be able to balance or deſtroy any inequality in libration: as in both caſes the actions of the parts are the ſame, the word applies both to the arch of equilibration, and to the arch enabled to ſtand by virtue of the mode in which the parts are connected.

Every arch may be an arch of equipollence if the actions of the parts be equal. Equal actions may be obtained by parts of equal ſizes, and of proportional ſpecific gravities; or of different ſizes, and the ſame ſpecific gravity, as in an arch of equilibration; or of equal

ſizes,

ſizes, and the ſame ſpecific gravity, and faſtened as in a catenaria; or of any ſizes, and any ſpecific gravity, but ſecurely faſtened, as in arches which are built rather from experience than principle.

Here it is to be obſerved, that the tenacity of the parts, or the coheſion, is always a reſiſting power; and from this circumſtance an arch may be doubly ſtrong: it may be equipollent from both equilibration and faſtening.

In our endeavours to form a perfect arch, the conſideration, that the more nearly we attain equilibration, the nearer we approach to perfection, is of the utmoſt importance.

An arch of equilibration being able to ſtand of itſelf will be more able to ſuſtain an additional weight, and the whole reſiſtance of the coheſion of its parts will oppoſe that additional weight; whereas in an arch, which does not equilibrate, part of the coheſion will be employed in reſiſting the inequality in the actions of its own parts.

Equilibration in the baſis is as important as in the ſtructure of the arch. When a foundation is uniformly ſoft or compreſſible, it is better and cheaper to erect the building on a planked grating or ſleepers, allowing an extra height for the probable ſinking. When the foundations are not uniform, an equal bed muſt be obtained by excavation or piling.

As no weight has any horizontal force, and the reſiſtance which a force, acting in a horizontal direction, meets with from a body lying on a horizontal plane, is cauſed by the coheſion between that body and the plane; no weight, therefore, however ſmall, can be ſuſpended between two powers acting in a ſtraight line, the parts being perfectly free to move.

Fig. 1. If A were a lubricous particle, and F A a lubricous horizontal plane, no lubricous weight could reſiſt the force of A in the direction A F; two infinite powers muſt be therefore obtained by the depth in which the foundations are ſunk in the earth.

All bodies lying on level beds, which receive a lateral impulſe, ſhould have as much coheſion and friction as can be given them.

Coheſion and friction are increaſed by the weight of bodies, or by cement and faſtening.

A catenarian curve, or the line formed by a perfectly flexible chain, ſuſpended at its two extremities, is an arch of equipollence.

It

It has hitherto been aſſerted, that if this line were inverted, and compoſed of infinitely ſmall rigid and poliſhed ſpheres, it would retain the ſame form: from this it has been concluded, that an arch of equal vouſſoirs, forming this curve, would be an arch of equilibration.

The fallacy of this opinion appears manifeſt, as the connexion which equalizes the action in one caſe is of no uſe in the other; therefore it could not be even equipollent.

It is evident the connexion is neceſſary to equalize the actions, ſince bodies in inclined directions, whether ſupported or ſuſpended, loſe part of their gravity, according to their directions; and as the directions of the links in a chain differ, their actions muſt be unequal, unleſs ſome other force except gravity be applied to equalize them. This error ſeems to have ariſen from confounding the equipollence of a catenaria with equilibration.

Extract from Dr. David Gregory's paper on the Catenaria, Phil. Tranſ. 1697. *Fig.* 23.

" In a vertical plane, but in an inverted
" ſituation, the chain will preſerve its figure
" without falling, and therefore will conſtitute

 " a very

" a very thin arch, or fornix; that is, infinitely
" ſmall rigid and poliſhed ſpheres diſpoſed in an
" inverted curve of a catenaria will form an
" arch: no part of which will be thruſt out-
" wards or inwards by other parts, but, the
" loweſt parts remaining firm, it will ſupport
" itſelf by means of its figure. For ſince the
" ſituation of the points of the catenaria is the
" ſame, and the inclination of the parts to the
" horizon, whether in the ſituation A C B, or
" in an inverted ſituation, ſo that the curve
" may be in a plane which is perpendicular to
" the horizon, it is plain it muſt keep its figure
" unchanged in one ſituation as the other.
" And, on the contrary, none but the cate-
" naria is the figure of a true legitimate arch,
" or fornix. *And when an arch of any other*
" *figure is ſupported, it is becauſe in its thickneſs*
" *ſome catenaria is included.* Neither would it
" be ſuſtained, if it were very thin, and com-
" poſed of ſlippery parts. From Corol. 5. it
" may be collected, by what force an arch, or
" buttreſs, preſſes a wall outwardly, to which
" it is applied; for this is the ſame with that
" part of the force ſuſtaining the chain, which
" draws according to a horizontal direction.

" For the force, which in the chain draws in-
" wards, in an arch equal to the chain drives
" outwards. All other circumſtances, concerning the ſtrength of walls to which
" arches are applied, may be geometrically
" determined from this theory, which are
" the chief things in the conſtruction of edifices. Inſtead of gravity, if any other power
" exert its force, acting in like manner on a
" flexible line, the ſame curve will be produced. For example: if the wind be ſuppoſed equable, and ſhould blow, according
" to right lines, parallel to a given line, the
" line thus inflated by the wind would be the
" ſame as the catenaria. For ſince all things
" obtain in this other force, as we have ſuppoſed in gravity, it is evident the ſame line
" muſt be produced."

The truth of the ſentence in Italics, in the above quotation, is evident, though drawn from falſe premiſes. It is clear that two curves, whoſe parts are equipollent and free to move, whether from a relative connexion of the parts, or from equilibration, muſt produce the ſame form: but the ſame reaſon which is given, to prove that the catenaria would keep its figure unchanged in the one ſituation as the other, would apply to an arch of equilibration inverted, as its parts would have the ſame inclination to the horizon. When a chain forms a curve, and its parts are free to gravitate, they endeavour to aſſume the longeſt poſſible line; the curve formed by parts, which are in equilibration, and are free to gravitate, is the ſhorteſt poſſible line: in the one caſe mutual diſtention retains them in their ſituation, in the other mutual compreſſion. If the actions of the parts of an inverted catenaria be equal by gravitation, as they muſt be to retain their ſituation, then every joint in a chain is equally liable to be broken by the gravitation of the parts; but the contrary is evident from experience: therefore the inverted curve of a catenaria, compoſed of equal* rigid poliſhed ſpheres in a plane perpendicular to the horizon, cannot keep its figure. By mechanics the actions of

* Although Dr. David Gregory does not ſay equal, he evidently means it; and it has been ſo underſtood by others. —(Vide Dr. Hutton's Recreations in Mathematics.)

equal bodies, in different degrees of inclination, are unequal; but the actions of the links of a chain, free to gravitate, are equal by mutual opposition, and are in different degrees of inclination: therefore the absolute gravities of the links must be either proportionally increased by the opposition according to the inclination to render their actions equal, or the connexion, as a resisting power, must be a substitute for this increase. Now bodies may lose, but cannot acquire force by opposition, therefore it is the connexion which renders the actions of the links equal; and the inverted curve of a catenaria, composed of equal rigid polished spheres in a plane perpendicular to the horizon, without connexion, cannot keep its figure.

Fig. 2. Every particle of the arch B *c* B, A C A, is in an oblique direction, *a b*, *a b*, &c. except those in the direction *c* C, which are vertical.

The iron B *c* B is supposed inflexible, infrangible, and secure at the extremities B, B; it is evident, that if the material of which the arch is composed be not liable to be crushed or compressed, the arch cannot vary

in

in any part. To ſubſtitute an addition to the thickneſs of the arch for this ligature is a deſideratum in architecture.

It is evident that the deſtruction of the arch is to be attributed to the parts being forced upward, no part being able to fall until ſome other part has riſen to make way for it: the direction of the particles of an arch are conſequently from *b* to *a*, from C to *c*.

If the materials of an arch be liable to compreſſion, the iron B *c* B would become uſeleſs.

The object in uſing faſtening to the vouſſoirs of an arch, is to obtain an effect ſimilar to that which is produced by the iron B *c* B, and is ſimilar to the connexion of the links in a chain.

If every particle of the arch A C A be weighted, ſo that it can reſiſt the force with which the contiguous particles ſtrive to preſs it in the direction from *b* to *a*, however weak the material may be, it ſhall ſtand.

Perfection in machinery is precluded by cohesion and friction, becauſe motion, its object, is thereby impeded. In arches, ſtability being the object, cohesion and friction are auxiliary.

The

The larger the parts are in an arch, or, in other words, the fewer the joints, the leſs ſubject is it to variation, as the coheſion is greater: but the larger the parts, the more liable is the material to fracture.

Figures 3. and 4. A B C D are two blocks of ſtone, taken from a quarry in which they have been preſſed in the direction from B to D ever ſince their formation, ſo that their parts may be conſidered as linear, and ſimilar to the parallel lines A B and C D. Stone, in the direction from B to D, will bear almoſt any weight; but in the direction from B to A would be cruſhed or fractured by a comparatively ſmall one. It is evident that the vouſſoir E ſhall be ſtronger than the vouſſoir F, as the ſide *a b* partakes more of the frangible direction A B of the ſtone than *c d:* the nearer, therefore, the ſides of a vouſſoir aſſimilate to parallel lines, or the more vouſſoirs there are in an arch, the ſtronger will be the material.*

* The impropriety of making great ſkew-backs to ſtraight gauged arches is evident, as it reſpects the material; and it will be ſeen afterwards, as it affects the arch: it is a modern practice, and is ſeldom to be found in the old gauged arches.

If

If no aſſiſtance be required from cement, friction, and faſtening, but an equable action and reſiſtance in the parts be obtained by weight, the ſize of the vouſſoirs, the infrangibility, the incompreſſibility, if they be homogeneous, is of no importance. Cement, friction, and faſtening, and ſolidity of the material, do not at all deſtroy the equilibration of an arch; they, on the contrary, tend to reſiſt the uncertainty of preſſure from other cauſes, and are not to be neglected. A brick arch is compoſed of parallelograms, with the interſtices filled up with cement and ſlates, or other hard ſubſtances, or cement alone: the difficulty in practice of making theſe implements homogeneous with the bricks occaſions a ſettling.

Fig. 5. The intrados *a b* is ſuppoſed to equilibrate: the matter in the interſtices has the ſame effect as a horizontal force preſſing in the direction *e f* to keep the brick in the oblique direction *c d*; if it be inadequate, the brick turns on its point at the intrados, and every other brick follows.

Bricklayers from experience build their arches in ſeparate rings without bonding them

them together, except where the joints accidentally meet at *o*; by which method they diminiſh the ſize of the interſtice, and the arch of courſe approaches nearer to homogeneity, and the ſkill of the workman becomes leſs important.

Fig. 6. This arch is compoſed of ſtones, with parallel beds and perpendicular joints: it is evident no part of the arch can fall until ſome one of the ſtones *b b* ſlide outwards, or cruſh.

It is probable arches of this kind were anterior to the arches whoſe conſtituent parts have oblique lines of contact.

In this arch it will be ſeen, that the horizontal and perpendicular forces are reſolvable into oblique forces *c c*; and if the angles *c d c* were taken off, the intrados would be an arch.

A

TREATISE,

&c. &c.

SECT. I.

OF THE GENERAL LAWS OF MOTION, APPLICABLE TO THE SUBJECTS TREATED OF IN THE FOLLOWING WORK.

PROPOSITION I.

Fig. 7. IF two forces urge a body B at the ſame time in the directions of, and proportionally to, the ſides B A and B C of a parallelogram, it will move through the diagonal B D.

While B moves from B to C ſuppoſe the line B C to move parallel to itſelf into the line A D; the body muſt therefore be always in the moving line B C. When B C ſhall arrive at *a b*, let the body be arrived at *d*; then, ſince both the line B C, and the body B, move uniformly, the one along B A, and the other along B C, it will be as B *a* : *a d* :: B A : A D; therefore B *d* D is a right line.

1. The

1. The forces in the directions B A, B C, and B D, are respectively proportional to the lines B A, B C, and B D.

2. The two forces B A and B C may be compounded into the single force B D by drawing the diagonal of the parallelogram B D.

3. The single force B D may be resolved into the two forces B A and B C by describing a parallelogram, having B D as a diagonal, and B A and B C the directions of its sides.

4. If two forces, B A and B C, act in the directions B A, B C, respectively, draw B *e* to the middle of the right line A C, and twice B *e* is the force compounded out of B A and B C, and B *e* its direction.

Fig. 8. As the angle A B C shall decrease, B D will increase; and when the angle has vanished, the forces B A and B C will act together in the direction B D, and the compound force will be the greatest.

Fig. 9. As the angle A B C shall increase, B D will decrease; and when the angle has vanished, the forces B A and B C will act and react in a straight line, and there will be

no

no compound force. If the forces be equal they will balance or deſtroy each other; on the contrary, if they be unequal, the leſs of the two will be deſtroyed, and be moved in the direction of the greater with a force equal to the difference between them. Hence it is impoſſible to ſuſpend in a ſtraight line any weight, or a perfectly flexible chain, free to gravitate by the action of any forces, unleſs they act in a direction oppoſite to that of gravity.

PROPOSITION II.

Fig. 10. If two forces, A and B, ſuſtain a weight D, and the three forces equilibrate, they will be to each other reſpectively as the three ſides of a triangle drawn parallel to their lines of direction H F, F D, D H.

Let H D repreſent the weight, and produce A H and B H, and complete the parallelogram H F D E; and by the laſt propoſition the force D H is equivalent to the two forces H E and H F: put, therefore, the forces H E and H F inſtead of D H, and the forces will ſtill equilibrate. Therefore H F is equal to the oppoſite force A and H E, or D F to its oppoſite force B: therefore the three forces are reſpectively as H F, F D, and D H.

1. Hence the forces A B D are reſpectively as the three ſides of a triangle drawn perpendicular to their lines of direction, as ſuch a triangle will be ſimilar.*

2. The diagonal of a parallelogram muſt always be the line of direction of the reſiſting power; and the two ſides interſecting it muſt be in the direction of the acting powers.

* *Fig.* A. Demonſtration: If the line N D be at right angles to the ſide K M (as we ſuppoſe), we have the two triangles, M N D and M K P, alike; as they have each a right angle, and the angle K M P common: the angle K will be equal to the angle N D M. By a ſimilar reaſoning it will appear, that the angle L is equal to the angle O D M; but as the angle O D M is equal to the alternate angle E H D, it follows that the triangle K M L is ſimilar to the triangle E H D.

The power A may therefore be expreſſed by K M, the power B by M L, and the power C by K L, each line being at right angles to the power which it expreſſes.

It may be needleſs to obſerve, that any triangle, K R S having its ſides at right angles to the direction lines of the power, although they do not cut them, will alſo expreſs the relation of the powers A B C.

3. If

3. If any number of forces act against any point in one plane, and equilibrate, they may be all reduced to the action of three, or of two, equal and opposite ones.

PROPOSITION III.

Fig. 11. If one body acts against another body by any kind of force whatever, it exerts that force in the direction of a line perpendicular to the surface whereon it acts.

Let the body B be acted upon by the force A B. The plane C D, and obstacle E, hinder the body B from moving: divide the force A B into the two forces C B and F B, the one parallel to the plane C D, the other perpendicular; then the plane will receive the perpendicular force F B, and the obstacle E the parallel force C B. Take away the obstacle E, and the force C B will move the body along the plane C D with no other hinderance than may arise from cohesion, occasioned by the pressure of F B on B, which would be nothing, were the surface and body B perfectly smooth. The force C B, there-

 fore,

ſore, having no effect, the remaining force F B will be the only one whereby the body B acts againſt the plane C D, and that in the direction F B, perpendicular to it.

PROPOSITION IV.

Fig. 12. The force wherewith a rolling body deſcends upon an inclined plane is, to the force of its abſolute gravity, by which it would deſcend perpendicularly in a free ſpace, as the height of the plane is to its length.

Draw F E perpendicular to A B, (by Prop. II. *Fig.* 10.) A E (being horizontal, and conſequently at right angles to the direction line of gravity,) will expreſs the abſolute gravity of C; F E will expreſs the force with which it deſcends in the direction A B, being at right angles thereto; and A F, the force with which it preſſes the plane A B, being at right angles to the direction of that preſſure: and by ſimilar triangles, the weight of the body, its force in the direction of the plane, and its preſſure againſt the plane, are reſpectively as A B, D A, and D B.

Suppoſe

Suppofe the plane A B to be parallel to the horizon, the cylinder H will be at reft upon any part of the plane where it is laid; becaufe the fupport or hinderance from the plane is equal to the whole abfolute gravity, and its force is deftroyed, and there is no height to reprefent any.

If the plane A B be elevated, fo as to be perpendicular to the horizon, the cylinder C will defcend with its whole force of gravity, becaufe the plane contributes nothing to its fupport or hinderance.

1. If two bodies defcend by the force of gravity from the points D and B; one by the inclined plane B A, and the other by the line D A; and the former be equal to B A, and the latter to D A; they will require a fimilar force to fuftain them in their direction, or fhall defcend with equal force: or if there be two planes (*fig.* B.) of the fame height, and two bodies be laid upon them proportional to the lengths of the planes, they will mutually fuftain each other if they be connected by a ftring, having liberty of motion over a pulley, and acting parallel to the planes.

2. The gravity of a body is decreased the nearer its direction is to a horizontal line: if the direction be a horizontal line, it has no gravity.*

3. The difficulty of moving bodies on a horizontal line arises from the resistance of cohesion; and that of continuing the motion, from the resistance of friction.

4. In bodies on inclined planes, whatever be the length of the plane, if the vertical lines or heights be equal, the forces of the bodies (their weights being in proportion to the length of the inclined planes,) shall be equal.

PROPOSITION V.

Fig. 13. Let there be any number of lines, A B, B C, C D, D E, &c. all in the same vertical plane, connected together, and moveable about the joints A, B, C, D, &c. (the two ex-

* It will be observed, that matter can never lose its gravity but as two equal powers in opposition destroy each other; therefore the gravity of a body may be said to be diminished on an inclined plane, and to be destroyed, or the body be said to have no gravity on a horizontal plane.

treme

treme points, A and G, being fixed). It is required to find the proportion of the weights to be laid upon the joints B, C, D, &c. in the direction of the radii from those points, so that the whole may remain in equilibrio.

From the several joints, having drawn the lines B *b*, C *c*, D *d*, &c. representing the direction in which the weights are to act at the joints B, C, D, &c. erect the equal perpendicular lines B *n*, C *p*, D *d*, &c. and draw the horizontal lines *n b*, *p c*, &c. interfecting B *b*, C *c*, &c. in *b*, *c*, &c.; then B *b*, C *c*, D *d*, &c. will be proportional to the weights required.

TABLE I.

Table shewing the respective absolute weights of bodies sliding down inclined planes in the different degrees of inclination, between a horizontal and perpendicular line, so that their forces may be equal.

Degrees.	Absolute Weight.	Degrees.	Absolute Weight.	Degrees.	Absolute Weight
Horizontal.	Infinite.				
1	58,823	31	1,941	61	1,143
2	28,572	32	1,887	62	1,133
3	19,231	33	1,835	63	1,122
4	14,286	34	1,789	64	1,112
5	11,494	35	1,745	65	1,104
6	9,615	36	1,701	66	1,095
7	8,196	37	1,661	67	1,087
8	7,194	38	1,623	68	1,079
9	6,410	39	1,590	69	1,071
10	5,747	40	1,555	70	1,064
11	5,236	41	1,524	71	1,058
12	4,808	42	1,495	72	1,052
13	4,444	43	1,466	73	1,046
14	4,132	44	1,439	74	1,041
15	3,861	45	1,414	75	1,035
16	3,623	46	1,391	76	1,031
17	3,425	47	1,368	77	1,027
18	3,236	48	1,346	78	1,023
19	3,077	49	1,324	79	1,018
20	2,924	50	1,305	80	1,015
21	2,7[illegible]3	51	1,287	81	1,012
22	2,6[illegible]7	52	1,269	82	1,010
23	2,5[illegible]	53	1,252	83	1,008
24	2,457	54	1,236	84	1,006
25	2,364	55	1,221	85	1,004
26	2,283	56	1,206	86	1,003
27	2,203	57	1,192	87	1,002
28	2,132	58	1,179	88	1,001
29	2,062	59	1,167	89	1,000
30	2,000	60	1,155	90	1,000

TABLE II.

Table shewing the force of a body, whose weight is 1,000, sliding down inclined planes in the different degrees of inclination, between a horizontal and perpendicular line.

Degrees.	Force.	Degrees.	Force.	Degrees.	Force.
Horizontal.	0				
1	17	31	515	61	875
2	35	32	530	62	883
3	52	33	545	63	891
4	70	34	559	64	899
5	87	35	573	65	906
6	104	36	588	66	913
7	122	37	602	67	920
8	139	38	616	68	927
9	156	39	629	69	934
10	174	40	643	70	940
11	191	41	656	71	945
12	208	42	669	72	951
13	225	43	682	73	956
14	242	44	695	74	961
15	259	45	707	75	966
16	276	46	719	76	970
17	292	47	731	77	974
18	309	48	743	78	978
19	325	49	755	79	982
20	342	50	766	80	985
21	358	51	777	81	988
22	375	52	788	82	990
23	391	53	799	83	992
24	407	54	809	84	994
25	423	55	819	85	996
26	438	56	829	86	997
27	454	57	839	87	998
28	469	58	848	88	999
29	485	59	857	89	1,000
30	500	60	866	90	1,000

SECT.

SECT. II.

OF ARCHES OF EQUILIBRATION.

In the following propositions the sections of the arches and abutments are used for their solidities; and the parts are considered as homogeneous lubricous rectangular planes of equal thickness which present their edge in the section.

PROPOSITION VI.

Fig. 14. Given the length of one plane V *b*, and its direction; and the direction of other planes *e d*: the extremities *d d*, &c. of all the planes are tangents to the semicircle A V B, and compose the semicircle. The planes are rectangular, and of equal thickness; consequently their masses will be as their lengths.

Required the lengths *e d*, or the extrados *e e*, so that the actions at *d* may be equal; or that every part of the arch A V B may be in equilibrio?

Draw

Draw *dc* parallel, and equal to V *b* and C *e* horizontal, cutting *de* in *e, e* : *d e*, *d e*, ſhall be the lengths required.

Demonſtration: The actions of the weights *e d*, *e d*, are equal; as *c d*, *c d*, are equal: and A V B is an arch of equilibration, having *e e e* as an extrados.

1. If the planes were not at right angles to the tangents at the points *d d*, the lines *d e*, *d e*, &c. could not expreſs the maſſes of the planes, as *d e*, *d e*, would differ in their widths.

2. The plane *d e*, being at right angles to the tangent, muſt be the joint effect of the two equal actions of the adjoining planes, and the three forces ſhall equilibrate.

3. If the arch had any ſolid abutment of rock, or earth, *a e f*; the reſiſtance from *f* A *d e* would be alone neceſſary, and *e f* would be the boundary on that ſide; or the rock might be cut in the form *e d* A.

In the erection of arches it is not only neceſſary to obtain a good foundation to ſtand upon, but it is alſo neceſſary to obtain a good foundation to puſh againſt.

4. It has been before proved, that no weight

weight free to gravitate can be ſuſpended by two finite forces acting in a horizontal line; from C towards A, and from D towards B. An infinite horizontal reſiſtance muſt interſect the line *e e* ſomewhere; that the arch may equilibrate: the ſolid earth or rock is ſuppoſed to be that reſiſtance; therefore in no caſe can the height of V, above the earth or rock, be equal to the abſciſſa of the arch, when *e e* continued ſhall be the form of the extrados.

5. It will be obſerved, that the planes are conſidered rectangular parallelograms; conſequently, when placed in the circumference of a circle, there would be interſtices between them, increaſing in width from *d* towards *e*, were the arch only compoſed of planes iſſuing out, and having extremities in the intrados. But this is not the caſe, every interſtice as it occurs being filled with ſucceſſions of rectangular parallelograms; the extremities of which form intradoſſes to other arches of equilibration, until the whole ſpace *e d*, *e d* is filled: and the contact of every plane can be but at its lower angles.

6. It

6. It is held by ſome, that an arch which ſprings from a horizontal bed, C A, exerts no thruſt on the pier which ſuſtains it; and by others, that it does: the former is theoretically, the latter experimentally, true. By Prop. iii. *fig.* 11. if A C were a lubricous plane, the arch A V would ſlide off the pier at the level A C, being alſo lubricous, however thick or ſlender it might be. The arch itſelf can, therefore, exert no thruſt on the pier; but as the two ſurfaces always cohere, either from the weight or cement, or from both jointly, the arch A V, in endeavouring to ſlide off the pier in the horizontal direction, is reſiſted by that coheſion, and affects the pier.

Figures 15. 16. 17. 18. 19. 20.—7. The ſame principles, by which the equilibration of the ſemicircular arch is found, will apply to every curve whoſe tangents, and the thickneſs at the vertex, are given; and the ſame procedure as ſhewn in this example, when applied to another curve with the above reſtrictions, will produce its extrados of equilibration.

8. The lower the arch at V E may be in proportion to its ſpan A B, and the greater the thickneſs at V *b* may be, the nearer will the extrados, immediately over A B, approach a horizontal line.

PROPOSITION VII.

Fig. 21. Given the horizontal line A B, being the extrados of an arch, whoſe parts are lubricous, and are ſuſpended between two lubricous abutments; the force of one of the parts, V *b*, being alſo given.

Required the intrados, ſo that the parts ſhall be in equilibrio?

It is evident that C D would be the intrados required; and C A, *e a*, D B, &c. would be the direction of the parts, were it poſſible to obtain a reſiſting abutment in a horizontal direction: or rather to ſuſpend A B D C, by a preſſure at right angles to D B and C A; the abutment muſt therefore be oblique. Let F and H be the abutments, and A *e* and B *e* their lines of obliquity, forming the ſame angle with a vertical line; protract

A *e*

A *e* and B *e* until they meet in E. From E, as a centre, deſcribe the arch *d d*, and take any number of parts *d*, *d*; through which draw the lines *a e*, *a e*, equal and parallel to V *b*; and draw through *d*, *d*, *c d f*, *c d f*, radiating from E.

1. If *c f*, *c f*, &c. were the directions of the planes, the moment liberty of motion ſhould be given them, they would turn on *d* as a centre, and fall in the direction *a e*.

2. If the extremities *c c* were held firm, they would either bend in the direction *c d e*, or all the parts below the line *d d d* would break off, beginning from *e* at the abutment,* as thoſe parts could receive no reſiſtance from the abutments, and the arch muſt fall.

3. If the extrados had been *b b*, any thing below *b d*, the arch muſt fall inſtantaneouſly, as it could not receive any ſupport from the abutments between *m d m*.

4. As *c f*, *c f*, are not ſimilar planes, their actions cannot be repreſented by *a e*, *a e*.

* See the arch over the gateway leading from Gray's Inn Square to Gray's Inn Lane.

5. The

5. The intrados of an arch, whofe extrados fhall be horizontal, muft be horizontal alfo, and vice verfa; and the parts muft alfo be in a vertical direction, which is impoffible: and no arch of equilibration can have either a horizontal extrados or intrados.*

PROPOSITION VIII.

Fig. 22. Given the extrados C *b* D, and intrados A V B, being a femicircle to which the extrados is parallel.

Required the direction of the parts, fo that they may be in equilibrio?

* Where the fofites of arches, over the openings of windows, &c. are required to be horizontal, the arches are of this defcription, although thefe arches appear to bear great weight, in reality they only fuftain their own. They are (in brick-work) only four inches thick, and the inner part of the wall above is fupported either by a wood lintel, or an arch of a different defcription; and they are relieved from the part ftanding on them by laying the firft courfe of bricks all headers: by which means the weight is thrown upon the inner arch or lintel.

The parts of thefe arches are held together by the adhefion and infrangibility, and fhould be called brick or ftone fummers, rather than arches.

The

The force at V muſt be V *b*. Draw from any points, *d*, *d*, the lines *c d*, *c d*, parallel and equal to V *b*, and *c e* horizontal, cutting the extrados in *e e*; draw *e d*, *e d*, which are the direction of the parts having equal actions.

Referring to the laſt propoſition of the horizontal arch, it may be obſerved, that a fracture would alſo take place at this intrados, the part *h d* A of every plane being ſubject to be broken. The plane *d e* preſſes on the adjoining plane in the direction *d h*, and is reſiſted in an oppoſite direction from the abutment; and the part *d h* A can only be preſſed by a force equal to that with which it preſſes, therefore there is nothing to break it. Were the preſſure at the extrados, the parts *e m* would evidently be fractured from every plane, as there is no reſiſtence to the force with which it would be preſſed between *e* and *m*.

The extremities of the planes *e d*, *e d*, &c. if rectangular, cannot form the intrados of the ſemicircle; becauſe, if they form the intrados, they will be rhomboidal and diſſimilar, and being diſſimilar, *c d*, *c d*, cannot represent their respective forces: and C *b* D cannot be an extrados of equilibration to A V B.

When the extrados and intrados, of a composition of lubricous forces in equilibrio in the same plane, are parallel, they must be right lines; as the direction of each of the forces must form the same angle with a horizontal line: no arch of equilibration, therefore, can have its intrados and extrados parallel.

From this section it will be seen, that the extradosses of equilibration, of circular, elliptical, and cycloidal arches, are the most convenient forms for the extradosses of bridges: and thus nature, in this instance, has not been so much at variance with herself, as by her laws to prescribe a form to the extrados of a perfect arch,* which is wholly inapplicable to the uses of mankind.

As the directions of the parts of any curve (being respectively at right angles to the tangents of that curve,) must be more obtuse

* Vide Dr. Hutton's Principles of Bridges; and Emerson's Mechanics.

with

with the abſciſſa of the curve, as they are diſtant from the vertex, conſequently the extrados and intrados of the hyperbolic arch cannot approach each other; nor can thoſe of the parabolic keep always at the ſame diſtance: but all curves, compoſed of homogeneous materials, muſt be thicker at their extremities than at the vertices, to be in equilibration.

Theſe reſults are in oppoſition to thoſe of Dr. Hutton; but the methods, by the reſolution of forces, adopted here, are the ſame as in that work: the difference ariſes in the application.

It is manifeſt, that whatever weight be placed upon an arch will have a vertical direction, unleſs artificially arranged; and it is alſo manifeſt, that the direction of every part of an arch, except that at the vertex, is oblique. A weight acting vertically on a body rolling down an inclined plane, increaſes its action in the direction of the plane, and is merely an addition to the force of gravity; and the length of a plane is expreſſive of its abſolute weight when the height expreſſes its action in the direction of the plane: the ſame laws apply to the vouſſoirs of an arch. The object of inquiry, in our endeavours to obtain an arch of equilibration, is the abſolute weight of each vouſſoir in its oblique direction; whether ariſing from the materials ſolely of which the arch is compoſed, or partly from the weight incumbent: and it is always to be kept in mind, that when gravity is the ſole acting power in a compoſition of lubricous forces, that an additional force acting vertically, and not deſtroying the equilibration, cannot alter the directions of the parts; and, conſequently, that the equilibration of an arch muſt invariably depend upon equable actions in the directions of its vouſſoirs. It is, perhaps, not unneceſſary again to repeat, that the directions of the vouſſoirs ſhould be at right angles to the tangents of the reſpective parts of the intrados of the arch which they occupy.

In controverting the principles in the work above alluded to, this opportunity is taken of acknowledging the information which has been derived from it; and it is hoped, that other

other branches of architecture, which are but little underſtood, may alſo derive aſſiſtance from the extenſive mathematical knowledge which its author poſſeſſes.

SECT. III.

OF THE CATENARIA.

In inveſtigating the properties of the catenaria, it is neceſſary to conſider the means by which we acquire a knowledge of its form, and other circumſtances attending that mode of formation.

Fig. 23.—1. The line A B is formed from a chain compoſed of links connected together, having a free motion in any direction at each connexion, but inſeparable.

2. If one link is at reſt, the whole chain muſt be at reſt. If one link is in motion, the whole chain muſt be in motion.

3. When the links are free to gravitate,

they endeavour to aſſume the longeſt poſſible line.

4. The lines formed by a chain are either ſtraight or curved.

5. When the length of the chain D C is equal to the ſhorteſt diſtance between the extremities, and is free to move, the line is vertical.

6. When the length of the chain exceeds the ſhorteſt diſtance between the extremities of the chain forming the curve, and the links are free to move, the line A B, formed by the chain, is a curve, and the longeſt which the links can then aſſume; and is called the catenaria.

7. When a chain is free to move at every connexion, and forms a curve, a force acts at each extremity A B of the chain forming the catenaria, and there only; and when it forms a ſtraight line, a force acts only at one extremity D.

8. The line is drawn through the points of connexion.

9. No catenaria can be an oblique or horizontal ſtraight line, as no compound forces can

can ſuſpend a perfectly flexible weight in a ſtraight line.

10. The catenaria is multilateral,* the line from *a* to *d* through the link being a ſide: when the links are equal, and the line is ſtraight, the lines forming the catenaria are equal. When the links are equal, and the catenaria is curved, the line neareſt the centre of the earth is the ſhorteſt and horizontal, that link lapping moſt over the adjoining links; and the lines increaſe in length as they are diſtant from that line, and they approach nearer to vertical lines. If the forces were at an infinite diſtance from the ſhorteſt line, the two lines neareſt the forces would be vertical, and the longeſt.

11. Every ſide is a tangent to the curve; the direction of each link is at right angles to the tangent of the curve, or to the ſide made by the link in the curve.

12. Every link acts in a direction perpendicular to the line it forms in the catenaria; and when the ſum of the angles, which theſe di-

* The impropriety of calling the ſame figure both multilateral and a curve, is excuſed by a reference to the circle as a precedent.

rections form with a horizontal line, is equal to 180 degrees, the abſciſſa of that catenaria muſt be infinitely long.

13. "All catenarias are ſimilar to one "another, ſince they are generated by a like "conſtruction of like figures ſimilarly poſited. "Whence two right lines, alike inclined to the "horizon, drawn through the vertices of the "chains, will cut off ſimilar figures and por-"tions of the chains, which are proportional "to the right lines ſo cutting them off.

14. "If the chain A C B is ſuſpended at "the points A and E, which are at unequal "heights, the part of the curve A C E con-"tinues the ſame as if it had been ſuſpended "at the points A and B, which are equally "high; becauſe it is all one, whether the "point E be fixed to the plane or not."— Dr. David Gregory, Phil. Tranſ. 1697.

PROPOSITION IX.

Fig. 24. To deſcribe a catenaria. From any points *a a*, of a curve A B, erect any equal vertical lines *a e*, *a e*, and draw the indefinite horizonal lines *e c*, *e c*, which inter-ſect

ſect at *b*, by the lines *a b*, *a b*, at right angles to the tangents at thoſe points. Make *c b* equal to the line *e b*, and conſtruct the ſimilar triangles *c b d*.

Through *d d* draw the line required.*

Catenariæ may be of as many forms as the gravity and ſhapes of the links in a chain may be varied. The catenaria formed by ſimilar and equal links will be found from the circle, as in this propoſition.

* However ſimple this mode of conſtructing the catenaria may be, the reader is referred to Dr. David Gregory's paper, which will ſhow the thought and labour neceſſary for ſuch an inquiry. Although the author had early reaſon to doubt the univerſal opinion, that the properties of this line were excluſively within the reach of the higher mathematics, this method is nevertheleſs the reſult of various and repeated attempts.

SECT. IV.

OF ABUTMENT PIERS OF EQUILIBRATION; OF BRIDGES OF MANY ARCHES; AND OF THE FLYING BUTTRESSES OF CATHEDRALS.

PROPOSITION X.

Fig. 25.* BELOW the line E A B is ſolid immovable rock: upon which it is required to erect a ſemicircular arch, C V, &c. ſtanding on abutment piers, whoſe height is equal to C A from the line E A B, and the thickneſs of the arch at the vertex is equal to V *a*; ſo that the arch and abutment piers ſhall equilibrate in every part.

Deſcribe the extrados of equilibration *e e a*, to the arch C V, (by Prop. vi. *fig.* 14). By

* A pier is underſtood to be a body which has only a weight in a perpendicular direction to ſupport; as a poſt, or the middle ſupports in bridges where there are many arches.

An abutment pier is underſtood to be a body which both reſiſts a force in a perpendicular direction and in a lateral direction: in a bridge of one arch the piers are abutment piers, or in a bridge of many arches the external piers are abutment piers.

Prop,

Prop. ix. *fig.* 24. draw the catenaria *n n a*, cutting the line E A B in H. Draw the line H F, which interfect, in I; from I, as a centre, with the radius I H, interfect the extrados of the arch of equilibration in K. Draw K M and K H, and complete the parallelogram K H L M.

V M L H K is half the arch and abutment pier required: A N is the depth the abutment pier muft be funk in the rock. M L N is a filling-in to form the opening A V B. The direction of the parts, between M and V, converge to F; the remainder are parallel to K M. It is neceffary to obferve, that the catenaria itfelf might be the extrados of the arch; but then the direction of the parts muft be at right angles to the tangents of the intrados of that catenaria.

1. Although the part M C L N is of no abfolute ufe in fupporting the arch, or in refifting its fhoot, neverthelefs, in cafe any of the intenfities of the parts between K and H fhould be deftroyed by adventitious circumftances, it will be of important fervice as a refiftence.

2. It will appear by the experience of the buildings,

buildings, ſhewn in *figures* 26. 27. 29. and 30. that the joints of the maſonry in the abutment piers may be horizontal, the coheſion being a ſubſtitute for the oblique direction.*

By Prop. iv *fig.* 12. K M, *e m*, *a* V, &c. expreſs the force with which they are preſſed by a body ſliding down them, when *e m* expreſſes the action of the body in a horizontal direction. Now the reſiſtence which K M, *e m*, &c. can make to the body is infinite, admitting that its ſubordinate parts are incompreſſible; therefore there can be no limit to the weight which K M, *e m*, V *a*, &c. can ſuſtain, that weight acting at right angles to their directions: therefore, if the lubricity be deſtroyed, and the directions of the forces remain the ſame, the homogeneity of the material is of no importance. Hence ariſe two different modes of obtaining the extrados of an arch, which are the ſame in the reſult, (ad-

* A block of ſtone will be at reſt, lying on another block at an angle of 35 degrees: if, therefore, the true obliquity ſhall be leſs than 35 degrees, the joints may be horizontal; if more, then the joints muſt have an obliquity, or a ſubſtitute equal to the difference.

mitting lubricity as a datum) although ariſing from different applications of the reſolution of forces: the one is, as before deſcribed, by conſidering K M, *e m*, &c. expreſſive of the abſolute gravity; the other, by conſidering K M, *e m*, &c. expreſſive of the preſſure at right angles thereto. In either caſe the modes of conſtruction muſt produce an equable action; but it is of theut moſt importance that we ſhould conſider the latter in practice as the true and juſt method, and in all caſes to be reſorted to. By altering the homogeneity of the parts according to the firſt method, that is, by making ſome of the vouſſoirs of leſs ſpecific gravity, they would conſequently be protracted beyond the extrados *a k*; and the arch will, nevertheleſs, equilibrate, but the lower protracted parts muſt neceſſarily be unreſiſted, and be broken by the ſuperior. On the contrary, if *k m*, *e m*, &c. be conſidered expreſſive of the reſiſtence it oppoſes to the weight which it ſupports, the heterogeneity will not deſtroy the equipollence of the arch, unleſs the parts are lubricous. This latter method will not admit of any variation in the extrados: the one

one can never apply in practice, as no materials are lubricous; the other muſt apply invariably.

3. As the catenaria may be extended to infinity, ſo may the height of the arch and abutment piers. The greater the height of the arch, the nearer will the line K H approximate to a perpendicular; and if the height be infinite, C N continued ſhall be one ſide of the abutment pier, and a line parallel at an infinite diſtance from C ſhall be the other. In no other caſe can a perpendicular line form the boundary line of an abutment pier, containing the leaſt poſſible materials.

4. If any part of the abutment pier K H M L were under water, that part would loſe ſo much of its weight as might be equal to its bulk of water, admitting that the water preſſed on the underſide.

There are experiments in hydroſtatics to prove, that bodies ſpecifically heavier than water may be made to ſwim, by taking away the ſuperincumbent water; conſequently their ſpecific gravity, by immerſion, may be diminiſhed beyond what may ariſe from the whole

being

being immerſed, or the ſame effect will be produced.

It is alſo proved, that a body ſpecifically lighter than water may be made to ſink, or lie at the bottom of a veſſel, by preventing the water acting on the underſide: its ſpecific gravity will therefore be increaſed, and this diminution or augmentation of ſpecific gravity will be in proportion to the depth.

5. If K H, one ſide of an abutment pier, be immerſed in water from P to H, it will be preſſed by the water P R H, and its ſpecific gravity will be increaſed; becauſe, if it were to riſe, beſides the reſiſtence of its own gravity, it would have to remove the water P R H.—(See alſo the ſides E *r*, N *i*, *fig.* 28.)

6. If the piers of the arch were built upon ſtilts, or on ground through which water exudes or gulls up, it is evident the ſpecific gravity would be decreaſed by the water preſſing upon the under ſurface; but this preſſure could not have any effect on the equilibration or duration of the arch, as it is only an active uniform force inſtead of a reſiſting force.

7. In the erection of a bridge of many

arches,

arches, it is adviſable that each pier may be conſidered as an abutment pier; that is, that it may be able to reſiſt the thruſt of half an arch. If this be the caſe, if all the arches are of the ſame ſize, one centre will ſerve for the whole bridge.

Mr. Labely, in the erection of Weſtminſter bridge, from following this principle, made nine centres ſerve for the fifteen arches of that bridge. He turned the three middle arches firſt; he then ordered ſix centres for the arches on the Weſtminſter ſide, which afterwards ſerved for the arches on the Surry ſide.

8. *Figures* 26. and 27. are ſections of Trinity Church, Ely, and King's College Chapel, Cambridge.

The dotted lines ſhew how the abutment piers ought to have been conſtructed according to the foregoing theory: to contain the leaſt quantity of materials, and ſupport the arch independent of any thruſt from the roofs.

9. If an abutment pier ſhould be required for any arch of any curvature, it will be found by deſcribing a catenaria with the

ſeveral

ſeveral lines at right angles to the ſeveral tangents of the curve; in the ſame manner as the catenaria $n\,n$ was found to the ſemi-circle C V.

PROPOSITION XI.

Fig. 28. Given a river, of which A B is half the width: it is propoſed to build a bridge over it of three arches; the middle arch to riſe the height A V, above its springing A E, and the upper part of it to be a portion of a circle, whoſe radius is C V: its thickneſs at the vertex to be V F. Below the level G H is ſolid rock.

Required the form of the three arches, the thickneſs of the piers, the depth the piers ſhould be ſunk in the rock, and the extrados of the bridge; ſo that the whole may be in equilibrio?

From C deſcribe the indefinite circle V f, and find by Prop. x. *fig.* 25. the abutment pier $f\,\pi\,d$, &c. to the arch V f, having a given lateral reſiſtance at the level A B from the ſide arch. Continue $\pi\,f$ to C, interſecting A B in X; from X deſcribe the circle f E, touching V f at f. V f E is the form of the middle arch.

Find by Prop. x. *fig*. 25. the extrados and abutment pier F π z i β to the arch V E. With the radius X E deſcribe the indefinite arch N h, touching the line z i, whoſe centre R * is in the line A B.

E N is the thickneſs of the pier.

By Prop. x. *fig*. 25. find the extrados and abutment piers m γ, r n, to the arch N h, having V F as the thickneſs at the vertex, and the level G H as a lateral reſiſtance: the line γ r muſt neceſſarily be a tangent to the arch f E continued. Draw r I K i, and the pier is complete.

From B, on the line A B, ſet off the diſtance B T equal to R N; and from T, as a centre, deſcribe the indefinite arch B k. Find the centre † of the circle which ſhall touch

* R is found by drawing a line parallel to z i, at a diſtance X E, which will interſect A B at that point.

† This centre is found thus: Make V S equal to T B, and draw the line S T, which biſect in t. From t draw t w at right angles to S T, interſecting A C continued. The point of interſection is the centre required.

the arch V *f* at V, and the arch B *k*. Defcribe from it the arch V *k*, touching the arch B *k* in *k*; and by Prop. x. *fig.* 25. find the extrados of equilibration and abutment pier F W *o* M L to the arch V *x y k* B, whofe thicknefs at the vertex is V F, and having a lateral refiftance at the level G H. F W is the extrados of the bridge.

From the centre of the arch V *k* draw the line *e k* through T, interfecting the arch in *k*. Make N *h* equal to B *k*; and from *h*, through R, draw *h* P, interfecting *e k* in P. From P, as a centre, defcribe *h k*, touching the arches N *h* and B *k* in *h* and *k*.

N *h k* B is the fide arch required. The bridge is now complete.

1. By making E N an abutment pier, either of the arches may be turned, and the centre ftruck; and one centre will ferve for the two arches.

There are other great advantages and conveniences attached to this mode of conftruction; in refpect to damming out the water, conftruction of the centres, paffage of the river, &c. befides the important confiderations

 that

that the arches are independent of each other, and that the deſtruction of one arch will not be followed by that of the whole bridge; and that the middle arch may be turned firſt, by which the uſual ſettlements in the ſide arches may be avoided.

2. The pier E N is too wide by $\pi\ m$, when the bridge is complete, and every part has taken its proper bearing; generally it will be found, that the arch $\nu\ x\ y\ k$ will be too flat for the thickneſs V F at the vertex: the part $\theta\ \lambda\ \pi\ m$ is contrived to obviate that difficulty; and it does not deſtroy the equilibration of any part of the bridge, and is found thus. Let fall the perpendicular lines $\pi\ u$ and $m\ g$, and find the point in the horizontal line Y Z drawn through $\pi\ m$, which ſhall be a centre to the arch $\theta\ \pi$, touching V θ and the line $\pi\ u$,* and deſcribe the arch $\theta\ \pi$. Set off m Z equal to Y π, and deſcribe the indefinite arch $m\ \nu$ from Z as a centre. Find the centre b of an arch, which ſhall touch the arches $m\ \nu$ and V y,* and deſcribe the arch $\nu\ \lambda$.

* See the ſecond note in this propoſition: the centre is found in the ſame manner as ſhewn there.

3. From

3. From the foregoing it will be ſeen how important it is, that the thickneſs V F ſhould be properly conſidered; as from this the relation of the other parts of the bridge are determined.

4. Theſe principles may be applied with the ſame eaſe to the conic ſections and mixed curves, as to the circle.

5. The whole or part of the middle arch may be given, and the mode of conſtruction will remain the ſame.

When the number of arches in a bridge exceed three, or in the caſe of three arches, deſcribe any curve or mixed curve which may be beſt adapted to the deſired form of the extrados of the bridge, having A V for a ſemiconjugate, and A B for a ſemitranſverſe axis: the middle arch being given, all the others will be found from the touching of the curve V B.

PROPOSITION XII.

Fig. 29. Given the inſide of an abutment pier and bow I M L F, and the inſide of a pier

pier and arch D C P B, whofe thicknefs at vertex is B E: below K D is a folid foundation.

Required the extrados and abutment pier I K, and the thicknefs of the pier D N?

Draw P L, being a tangent to both the arches M L F and C B P, and the catenaria E R F H, (by Prop. ix. *fig.* 24.): the catenaria is interfected by P L at R, and the arch M L F at F. Draw F A and R G perpendicular. Either A D or G D may be the width of the pier D N. From D H fubtract the thicknefs of the pier N D, and the remainder will be the thicknefs of the abutment pier; and the extrados of the bow will be in proportion to that abutment.

By Prop. x. *fig.* 25. conftruct the extrados and abutment pier *b c d* to the arch V *a* W, having *a b* as a thicknefs at the vertex, and *d* the level of a folid foundation.

1. This figure is a fection of Salifbury cathedral, from Price's obfervations on that building.

2. It appears that the pier D N is inadequate as an abutment pier for the arch V W,

and

and is juſt adequate as a pier for the arch B C; but the architect found that it was prudent to widen it.—(See the ſets off *r* and *m.*)

The following quotation is from Sir Chriſtopher Wren's ſurvey of that cathedral.

" Almoſt all the cathedrals of the Gothic " form are weak and defective in the poiſe of " the vaults of the iſles: as for the vaults of " the nave, they are on both ſides equally " ſupported, and propped up from ſpreading, " by the bowes or flying buttreſſes, which " riſe from the outward walls of the iſles. " But for the vaults of the iſles, they are " indeed ſupported on the outſide by the " buttreſſes, but inwardly they have no other " ſtay but the pillars themſelves; which, as " they are uſually proportioned, if they ſtood " alone without the weight above, could not " reſiſt the ſpreading of the iſles one minute. " True, indeed, the great load above of the " walls, and vaulting of the nave, ſhould ſeem " to confine the pillars in their perpendicular " ſtation, that there ſhould be no need of " buttment inward; but experience hath " ſhewn the contrary. And there is ſcarce

" any

" any Gothic cathedral that I have ſeen, at " home or abroad, wherein I have not ob- " ſerved the pillars to yield and bend inwards " from the weight of the vaults of the iſle. " But this defect is moſt conſpicuous upon " the angular pillars of the croſs; for there, " not only the vault wants buttment, but " alſo the angular arches that reſt upon that " pillar; and, therefore, both conſpire to " thruſt it inwards, towards the centre of " the croſs."

3. From the foregoing it will be ſeen, that experience contradicts the theory which propoſes height as a ſubſtitute for abutments; as it is clear, however great the incumbent weight may be, no advantage is gained, except from the increaſed coheſion.

If the author's theory be true, the principles upon which Dr. Hutton propoſes to conſtruct the abutment piers, ſo that they may be in equilibrio with the ſhoot of their arches, are incorrect: arches and their abutment piers conſtructed according to the examples 1. and 2. (pages 63, 64, and 65, ſecond edition of his Treatiſe of Bridges,) could

could not ſtand a moment; and further, the thickneſs of the pier in the latter example ought to be leſs than that in the firſt.

In corollary the ſecond of the ſection of piers, in the ſame work, there is the following concluſion: " So that in this caſe it " makes no difference of whatever height " the pier is to the ſpringing of the arch; " for though the drift of the arch be in- " creaſed with the length of the lever, or " height of the pier, the weight of the pier " itſelf, which acts againſt it, is alſo increaſed " in the ſame proportion." And again, in the ſubſequent ſcholium: " In the fore- " going propoſition I have conſidered cir- " cular arches only, as it will make no " difference of any conſequence to ſuppoſe " the arches of any other curve of the ſame " ſpan or pitch."

To the former quotation the author oppoſes his own theory and the uniform practice in all abutments; and in reſpect to the latter, the form of curvature muſt neceſſarily make a very important difference in the ſize of the abutments, as

that

that ſize is governed by the tangents of the curve.*

The author would not have preſumed to have obſerved, as he has done, on the work above mentioned, or to have conteſted queſtions of ſo much difficulty with ſo eminent a mathematician as Dr. Hutton, had the ſubject been ſolely theoretical; but the difference between them is upon the application of the laws of mechanics to practice.

Fig. 30. is a ſection of Weſtminſter abbey. The catenaria *a b c* is drawn to ſhew how nearly the abutment piers correſpond with the principles laid down in the preceding propoſition. When there are double flying bows, as on the ſouth ſide of the abbey, the thickneſs of the ſecond piers may be found by drawing other catenariæ over the firſt bows, which will be interſected by the ſecond bows in a ſimilar manner to the firſt.

* The reader is referred to the Supplement to the third edition of the Encyclopædia Britannica, article Arch; and is requeſted to apply the obſervations generally upon arches, and particularly upon the common theory of equilibration with that which is here ſubmitted.

It

It will be ſeen alſo, that on the contrary, if the thickneſs of the piers be given, the ſituation of the flying bows may be found. The pillars A and B are evidently inſufficient for the arches of the ſide iſles, and have been found ſo in the building.

PLATE 13. is a ſection of the Chapter Houſe at Lincoln.* PLATES 14. and 15. are ſections of Lincoln cathedral. The architect employed in the erection of theſe buildings has not followed implicitly the principles by which the architects were governed who erected the other edifices of which ſections are here given. The ſituations and forms of the flying bows of the cathedral are not the moſt advantageous, and the abutment piers are ſomewhat deficient in ſubſtance; the walls of the nave have been conſequently thruſt outwards, and the vaultings ſtill continue that

* *Fig.* 31. in this plate, is a drawing compoſed of ſtone vouſſoirs, ſuſpended between the towers of Lincoln cathedral above the vaulting at the weſt entrance of the nave. The radius of this arch is about 91 feet. This arch is given to ſhew, that when the abutments are immoveable, how extremely thin an arch may be at the vertex.

action

action upon them. The buttreſſes to the Chapter Houſe, which are larger than neceſſary had the ſpringing of the bows been lower, are thruſt conſiderably out of a perpendicular, and one of the bows muſt be immediately rebuilt. The author ſuggeſted to an intelligent carpenter, who attended him, the height in the abutment from which the bow ſhould ſpring.*

The ambitious hopes of attaining ſtill further knowledge in the ſcience of building, evidently induced the artiſt who conſtructed theſe edifices to make the daring experiments in vaulting, which are there ſhewn. Upon a reference to dates, it will be found, that Saliſbury cathedral was erected ſoon after that at Lincoln: this circumſtance, and the analogy between them, indicate, that to the

* The author has obſerved in his examination of buildings, but particularly in cathedrals, that where irons have been introduced as ties, that they are ſeldom fixed in the moſt advantageous ſituations. As an inſtance, the propoſition on flying buttreſſes will ſhew, that the ſpringing of an arch, its tangent being a vertical line, is by no means the proper poſition for a tie, when the thruſt of the arch is too great for the thickneſs of the abutment.

experience

experience gained in the conſtruction of the latter, the former is indebted for much of the admiration it has occaſioned.

PLATE 16. is a ſection of Ely cathedral. PLATES 17. and 18. are ſections of Peterborough cathedral: the one is of the new building at the east end, and appears, from the ſtyle of architecture, to have been erected by the ſame architect who erected King's College Chapel, Cambridge. It is worthy of remark, that the roof of Peterborough cathedral is ſimilar in principle to that deſcribed to be in the fauxbourg Saint Honoré at Paris;* which, although without tie-beams, is ſtated to have no lateral thruſt on the walls on which it reſts. It appears, however, that the architect here entertained a different opinion, and builders in general will doubt the poſition.

PLATE 19. is a ſection of York cathedral. The vaulting over the nave is wood; but it was the intention of the architect that it

* See Montucla's Recreations in Mathematics and Natural Philoſophy; tranſlated by Dr. Hutton.

ſhould be stone, as the parts of the flying buttreſſes, the ſtone couſſinets, and the ſize of the abutments, evidently ſhew: the vaultings over the iſles are ſtone.

THE END.

T. Benſley, Printer,
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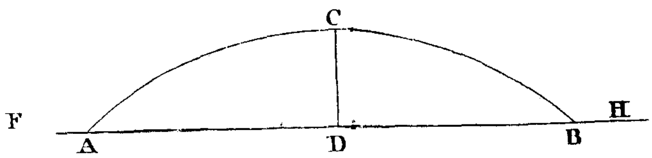
Fig 1.



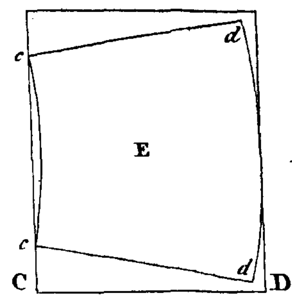
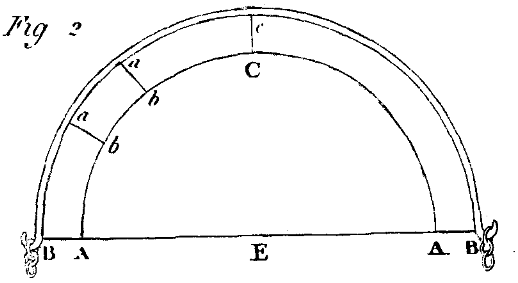
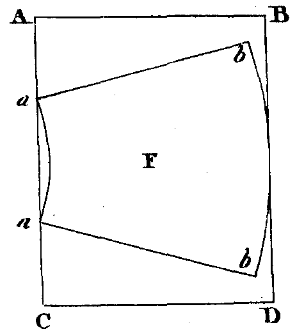


Fig. 3.

Fig 2





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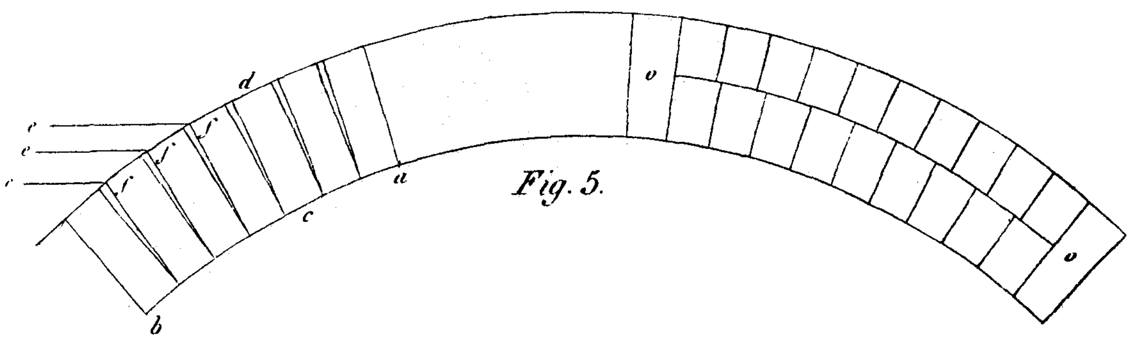

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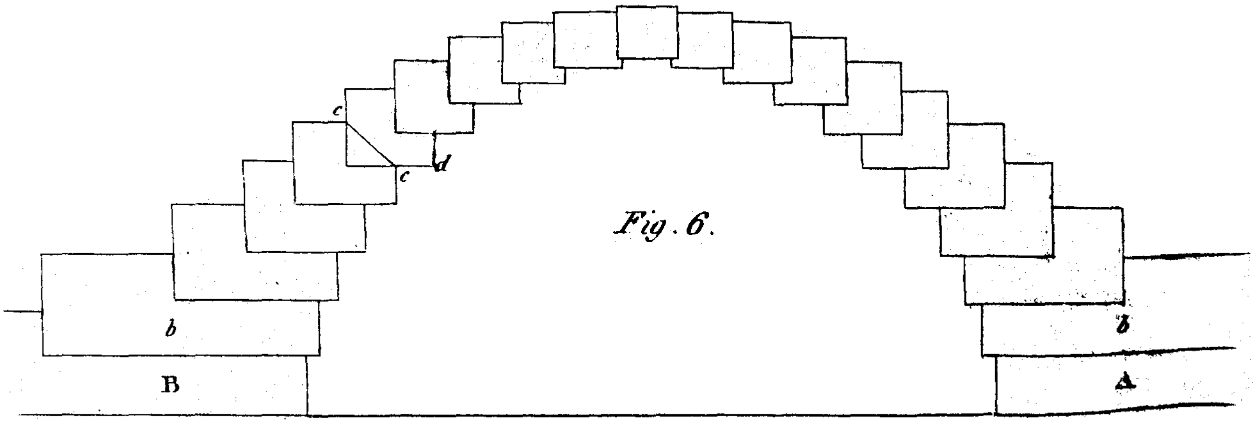
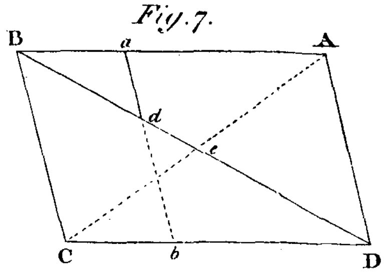


Fig. 6.

Plate

Fig. 7.



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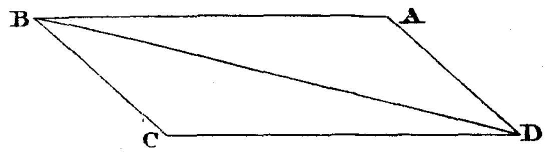

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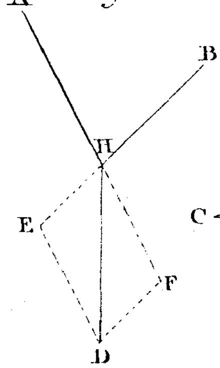

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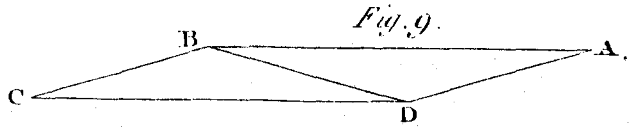


Fig. A

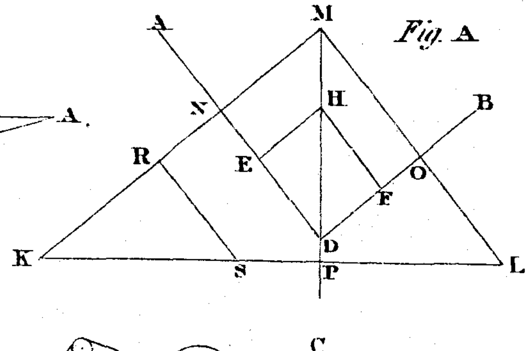
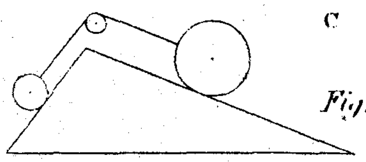


Fig. B.



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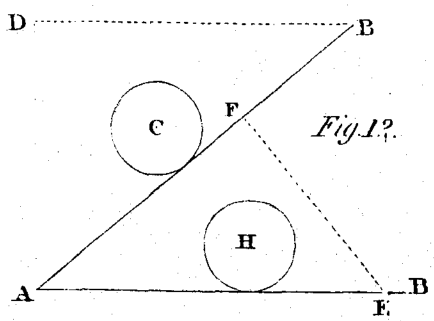


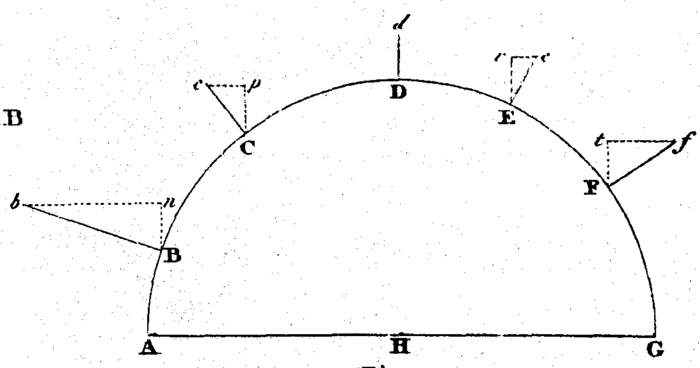


Fig. 13.

Fig. 11.

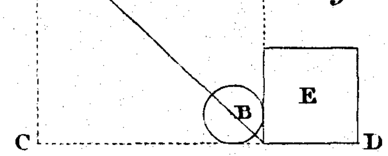


Plate .

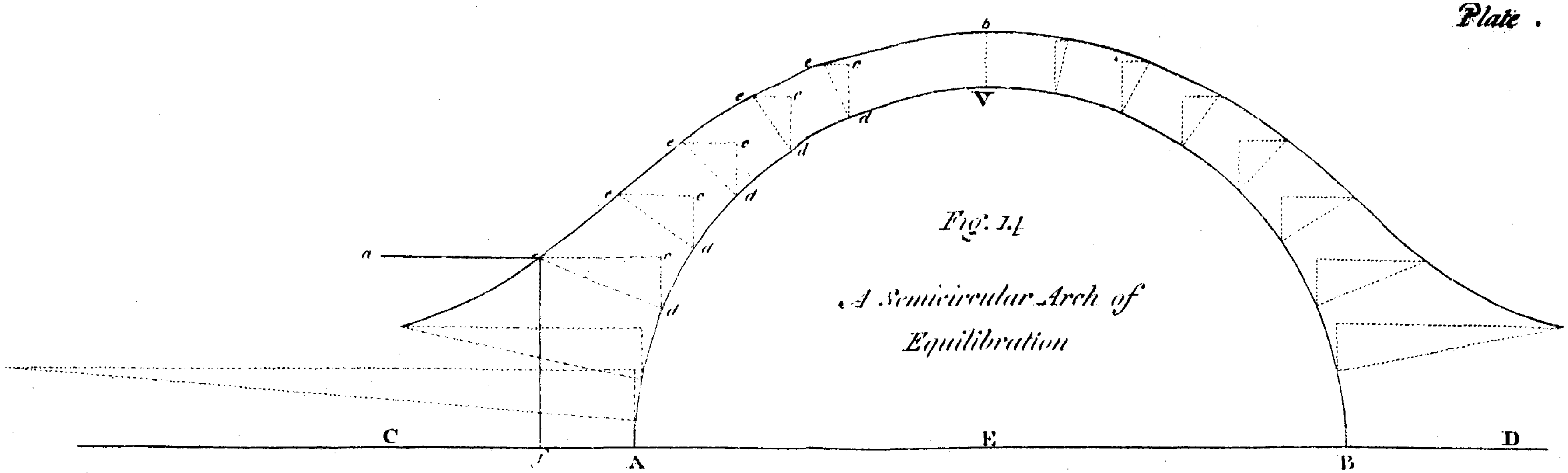


Fig: 14

A Semicircular Arch of Equilibration

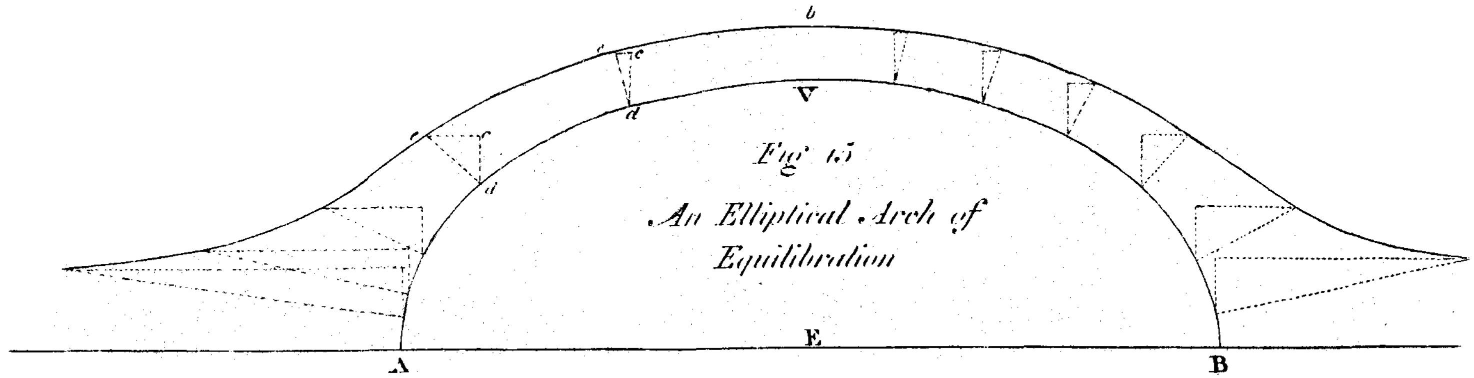


Fig: 15

An Elliptical Arch of Equilibration

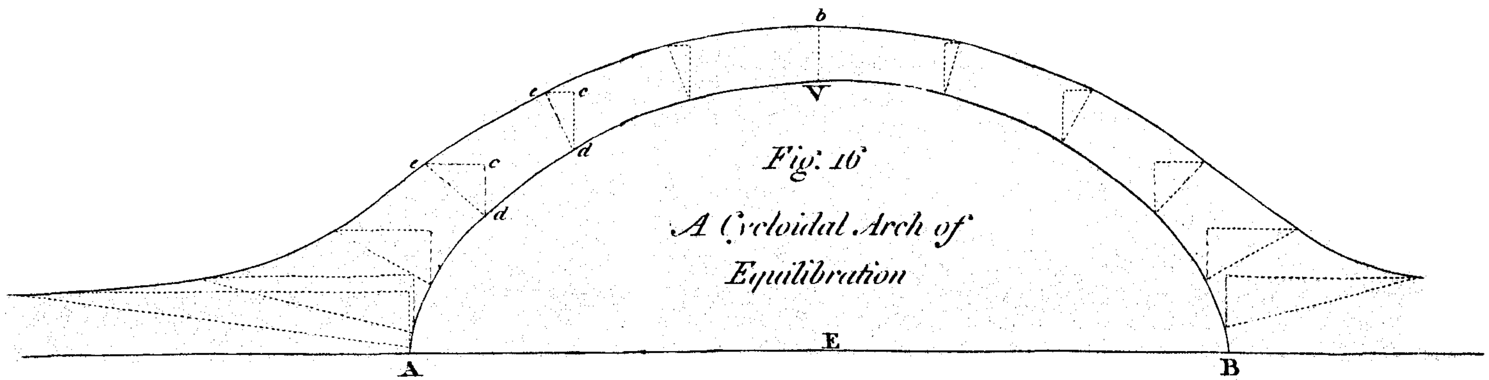


Fig: 16

A Cycloidal Arch of Equilibration

Neele sc. Str

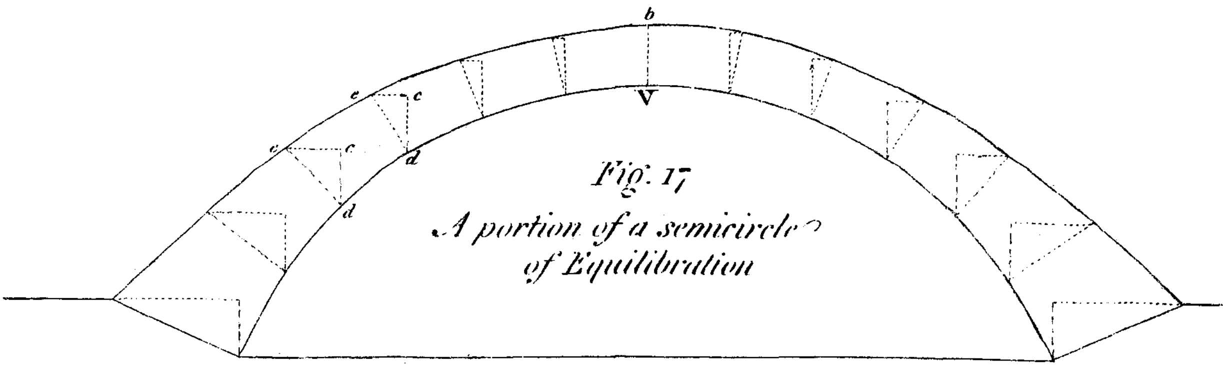


Fig: 17
A portion of a semicircle
of Equilibration

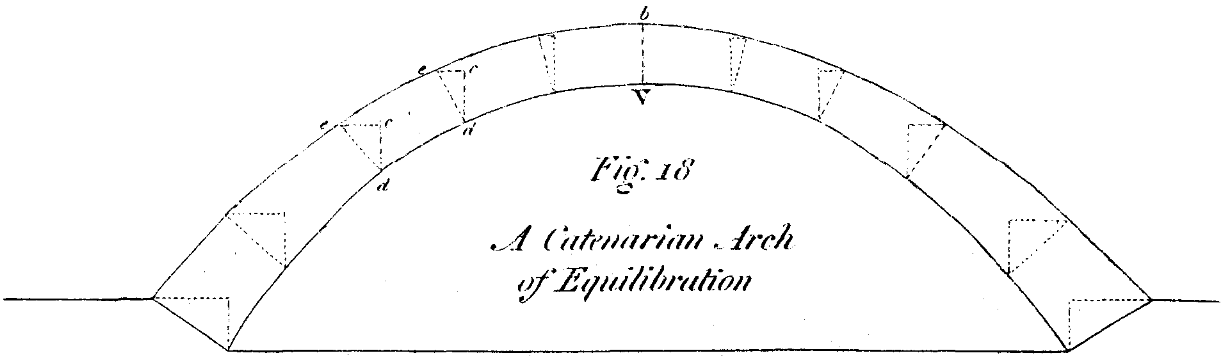


Fig: 18
A Catenarian Arch
of Equilibration

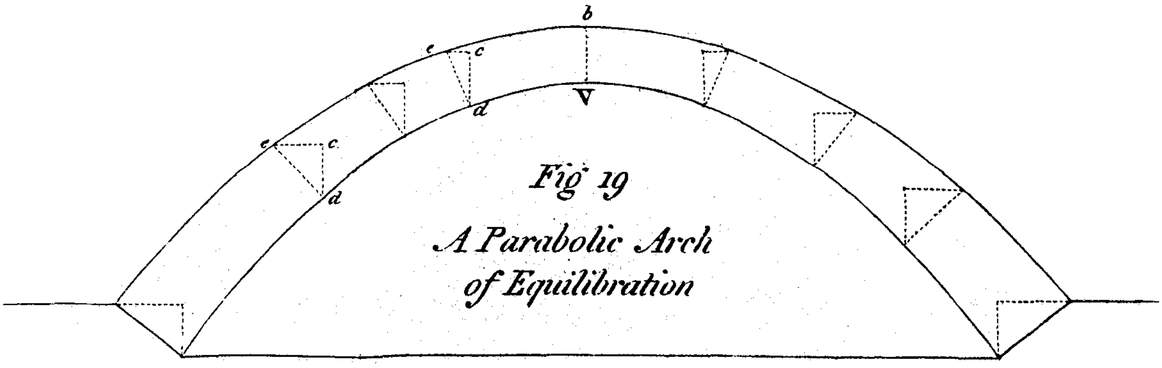


Fig 19
A Parabolic Arch
of Equilibration

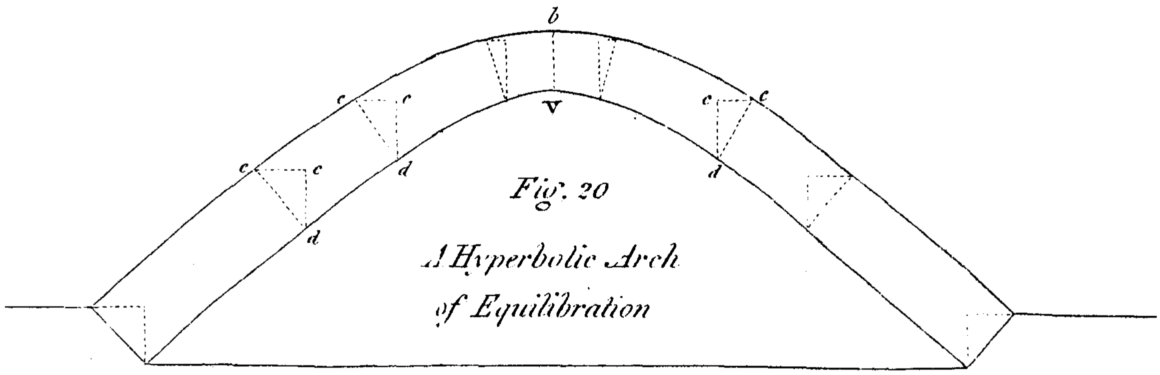


Fig. 20

A Hyperbolic Arch of Equilibration

Fig. 21.

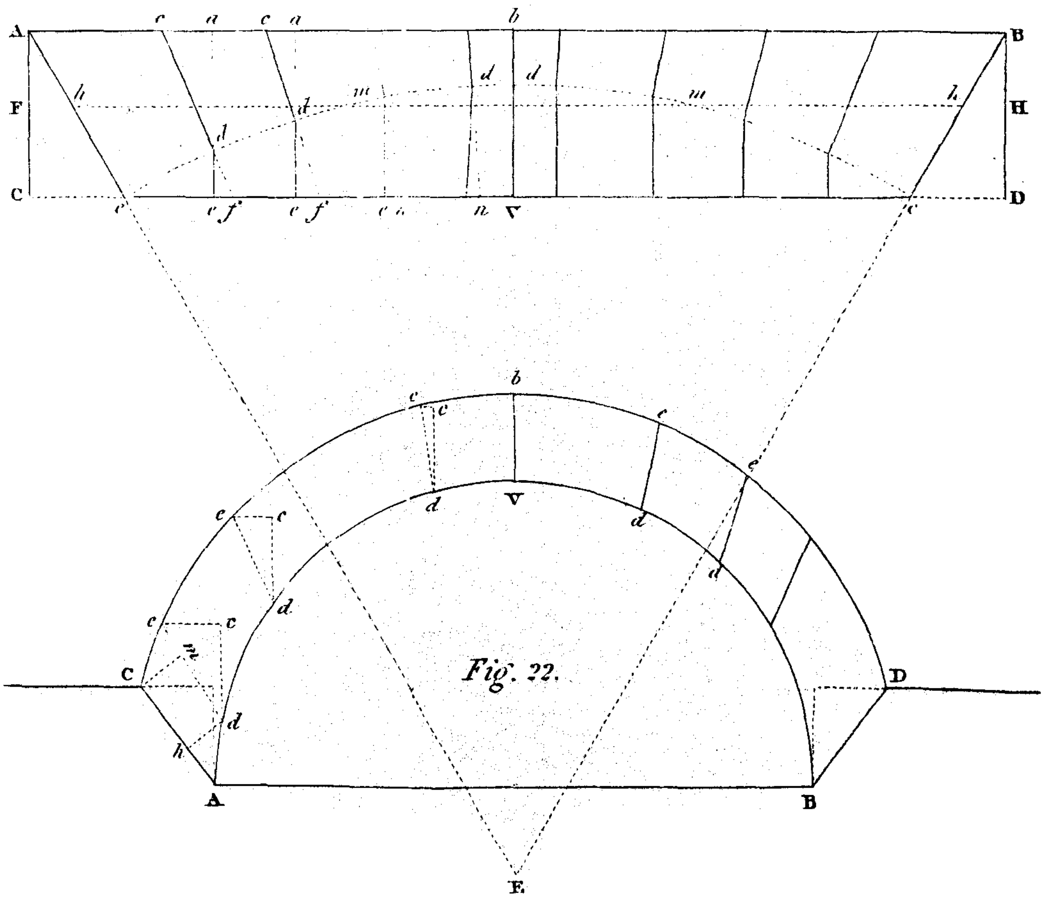


Fig. 22.

Neele sc. Strand

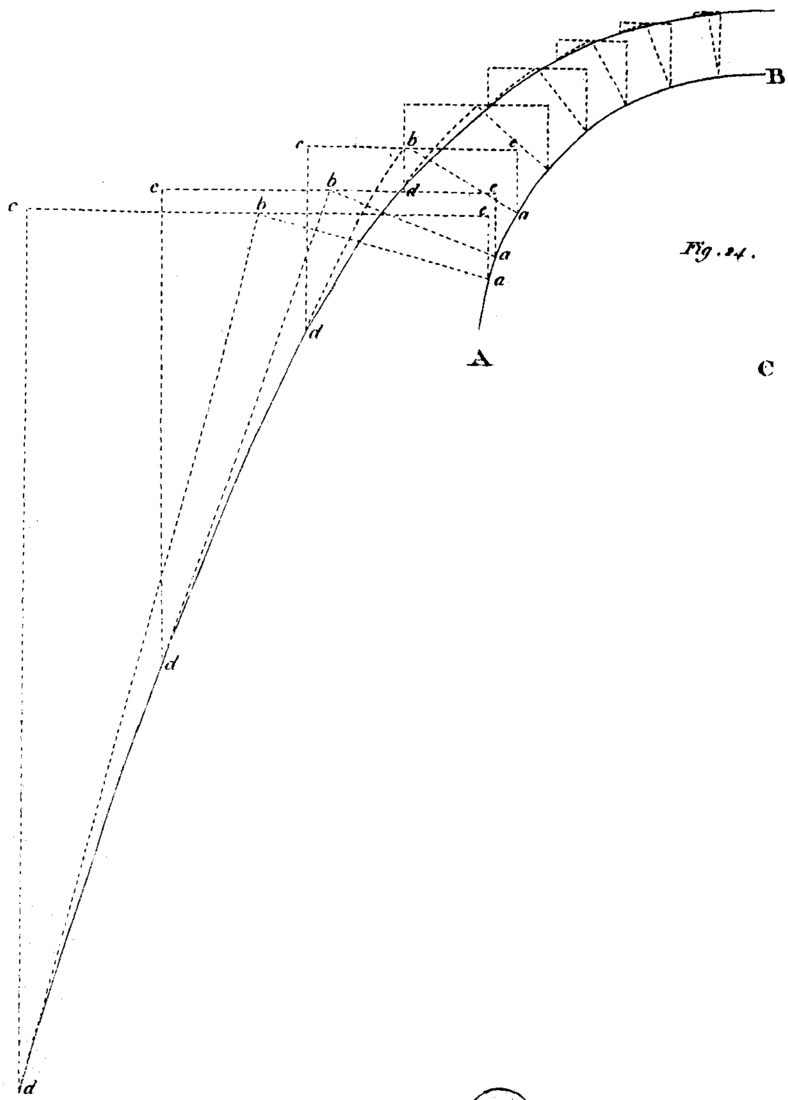
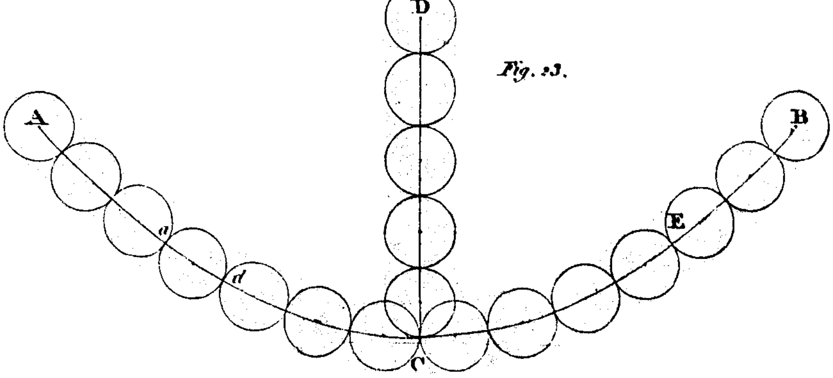


Fig. 24.



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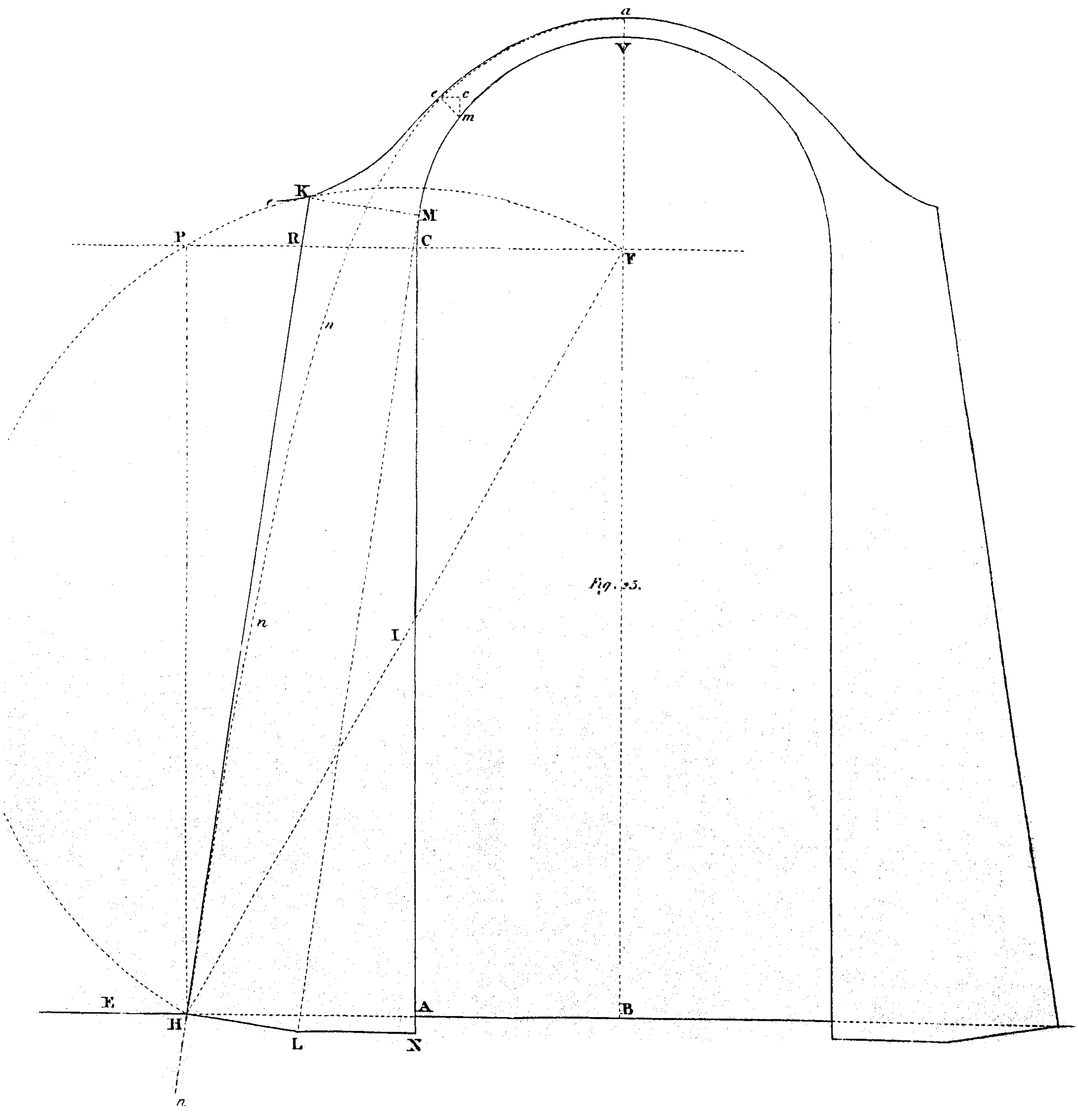
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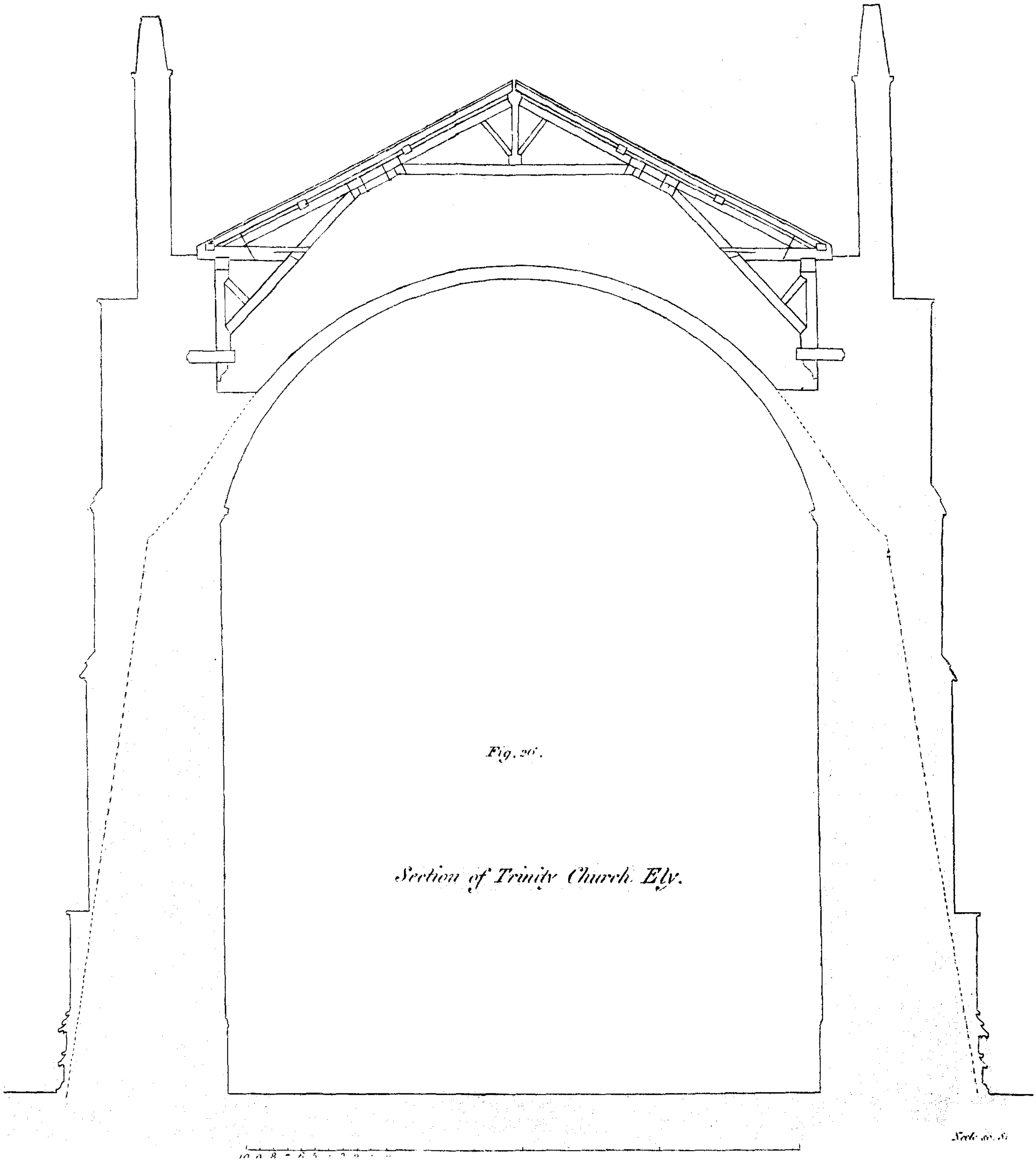


Fig. 26.

Section of Trinity Church Ely.

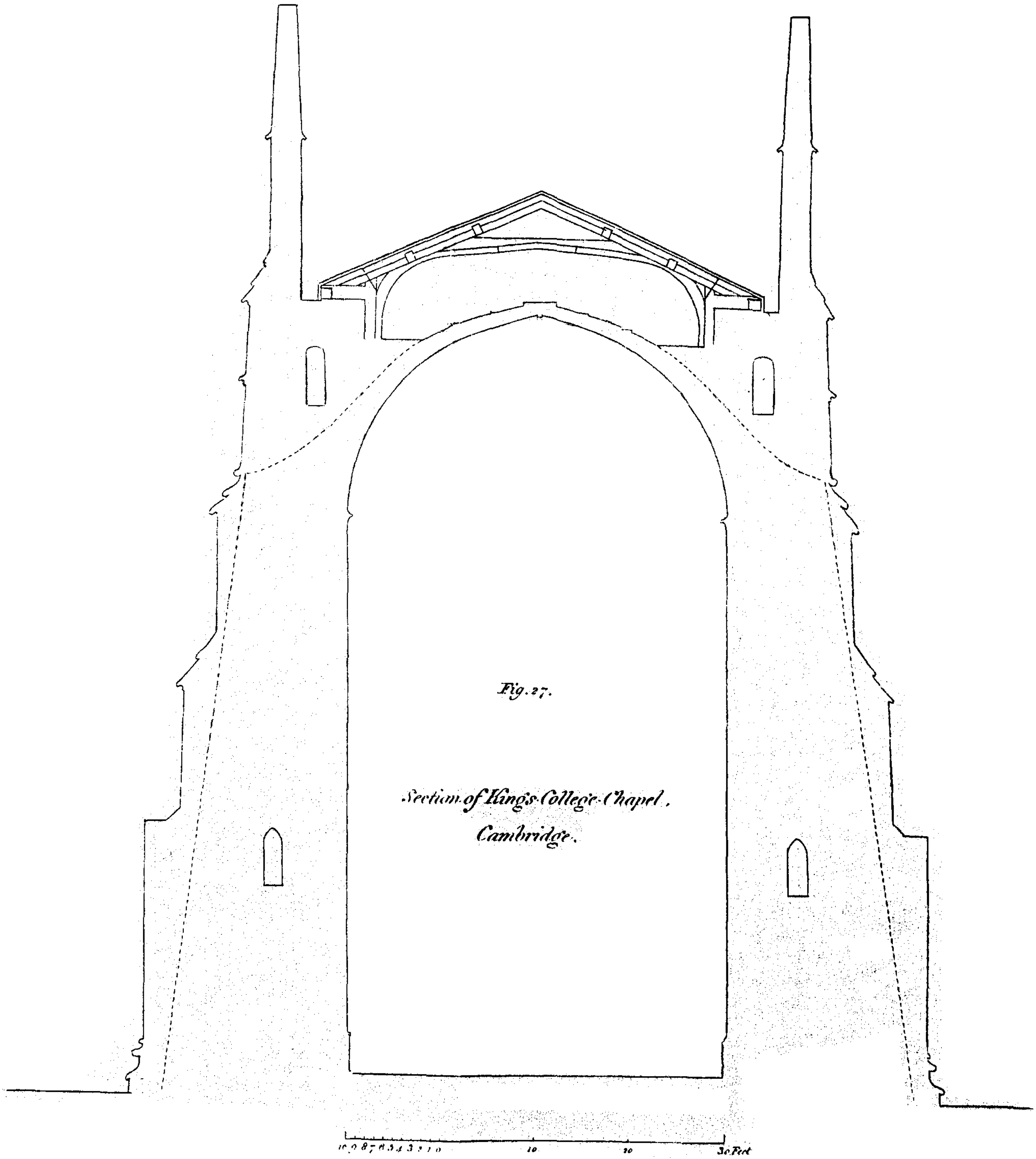
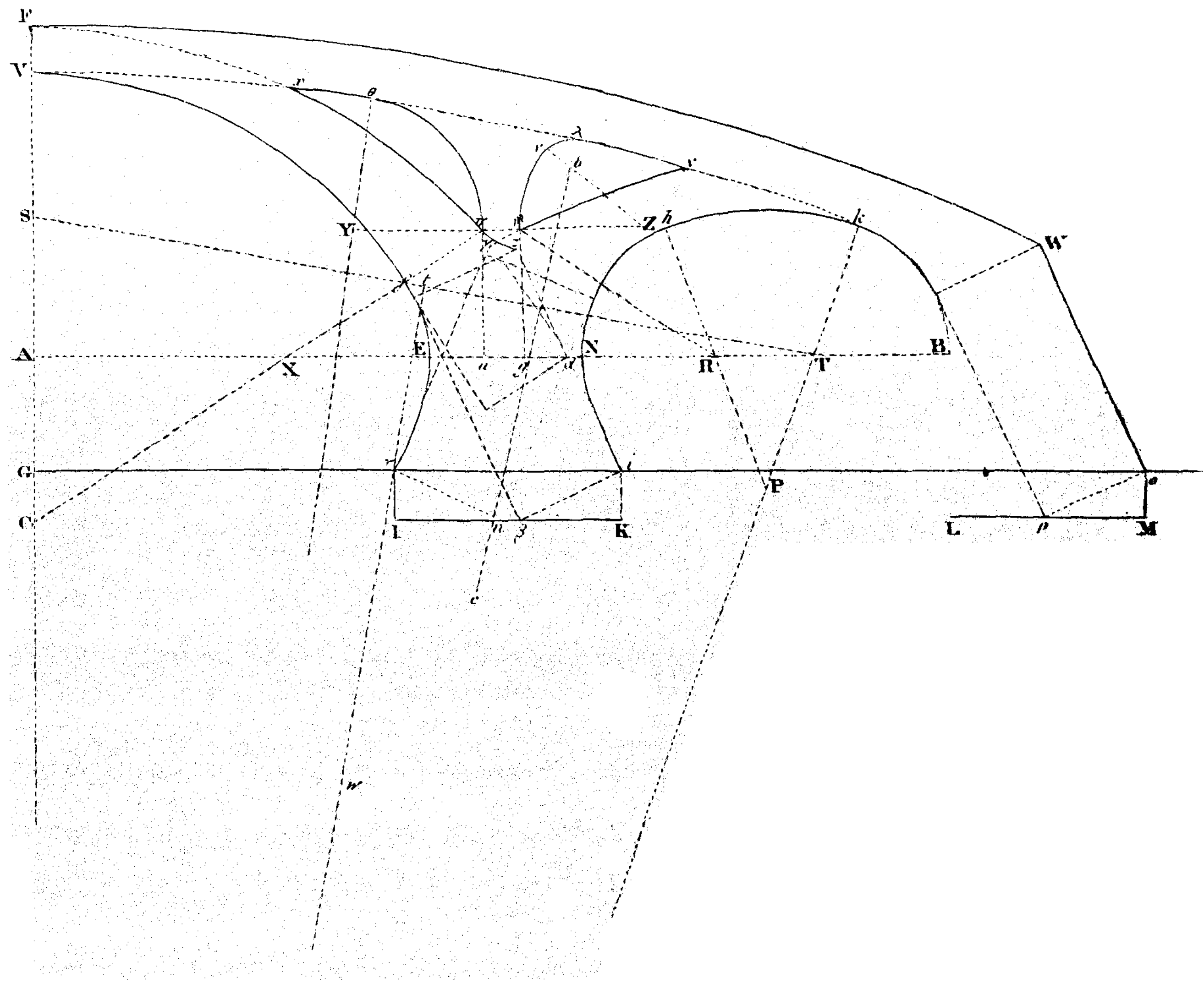


Fig. 27.

Section of Kings College Chapel, Cambridge.

Fig. 4.



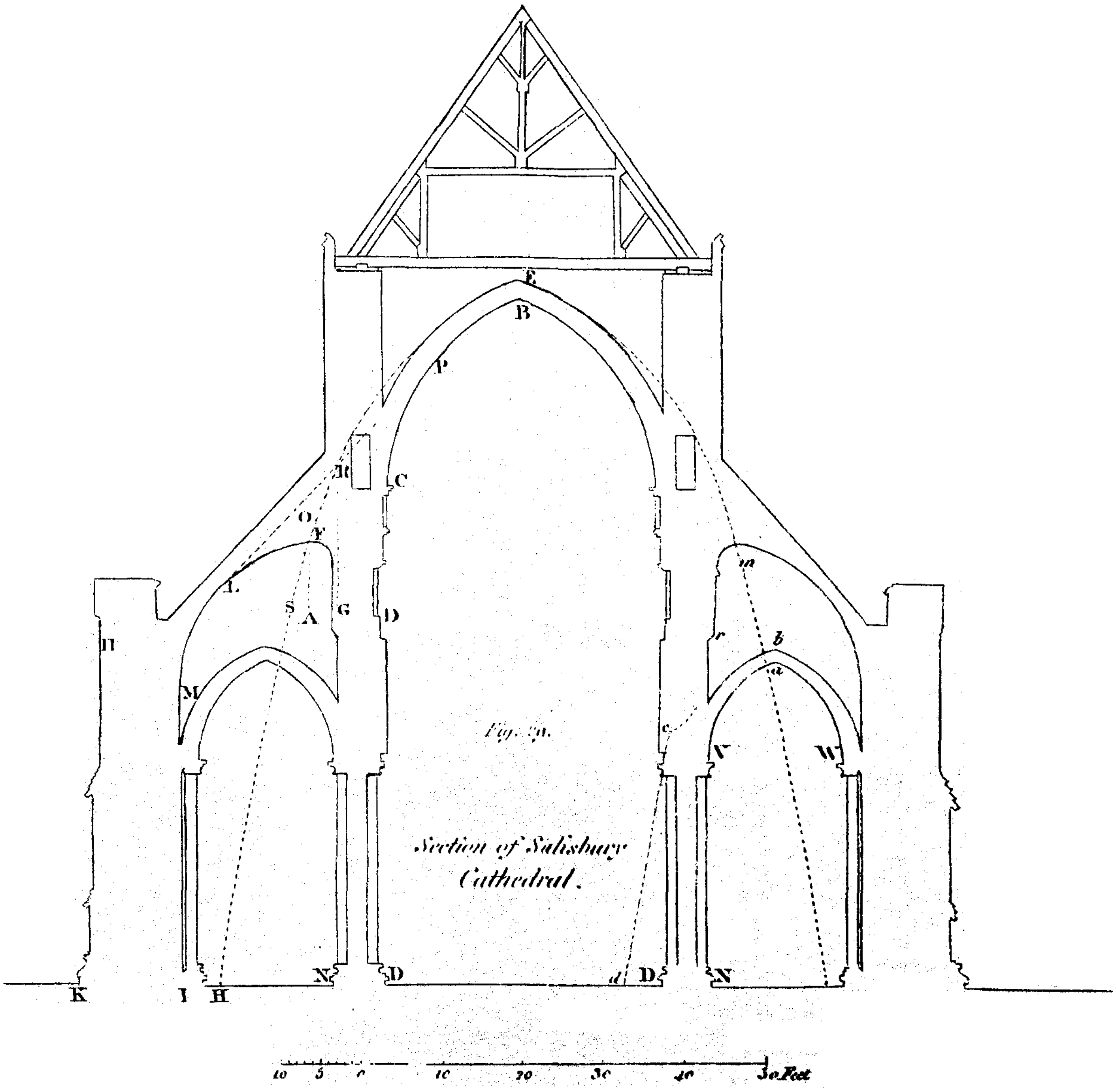


Fig. 71.
Section of Salisbury Cathedral.
10 5 0 10 20 30 40 50 Feet

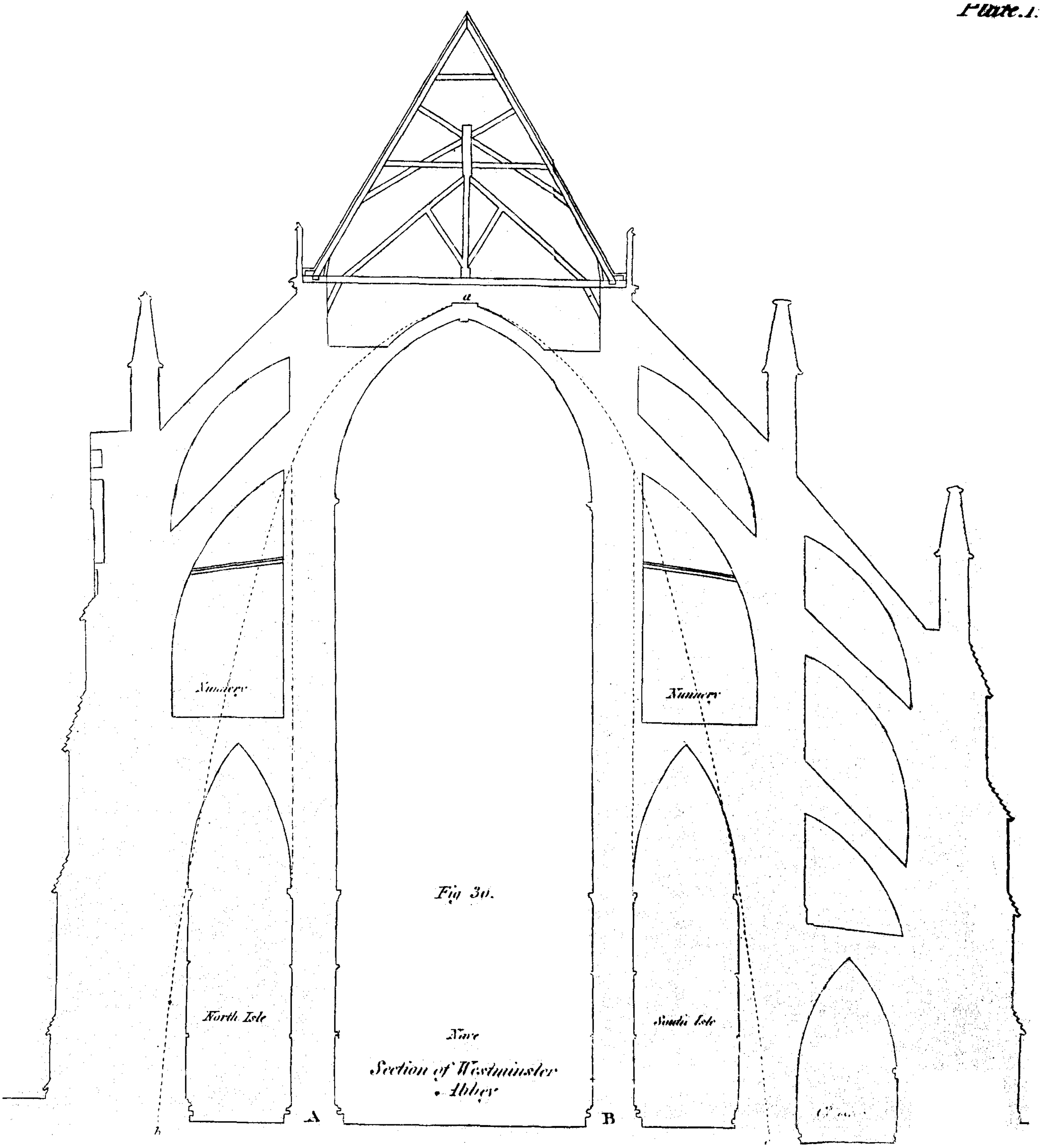


Fig 30.

Section of Westminster Abbey

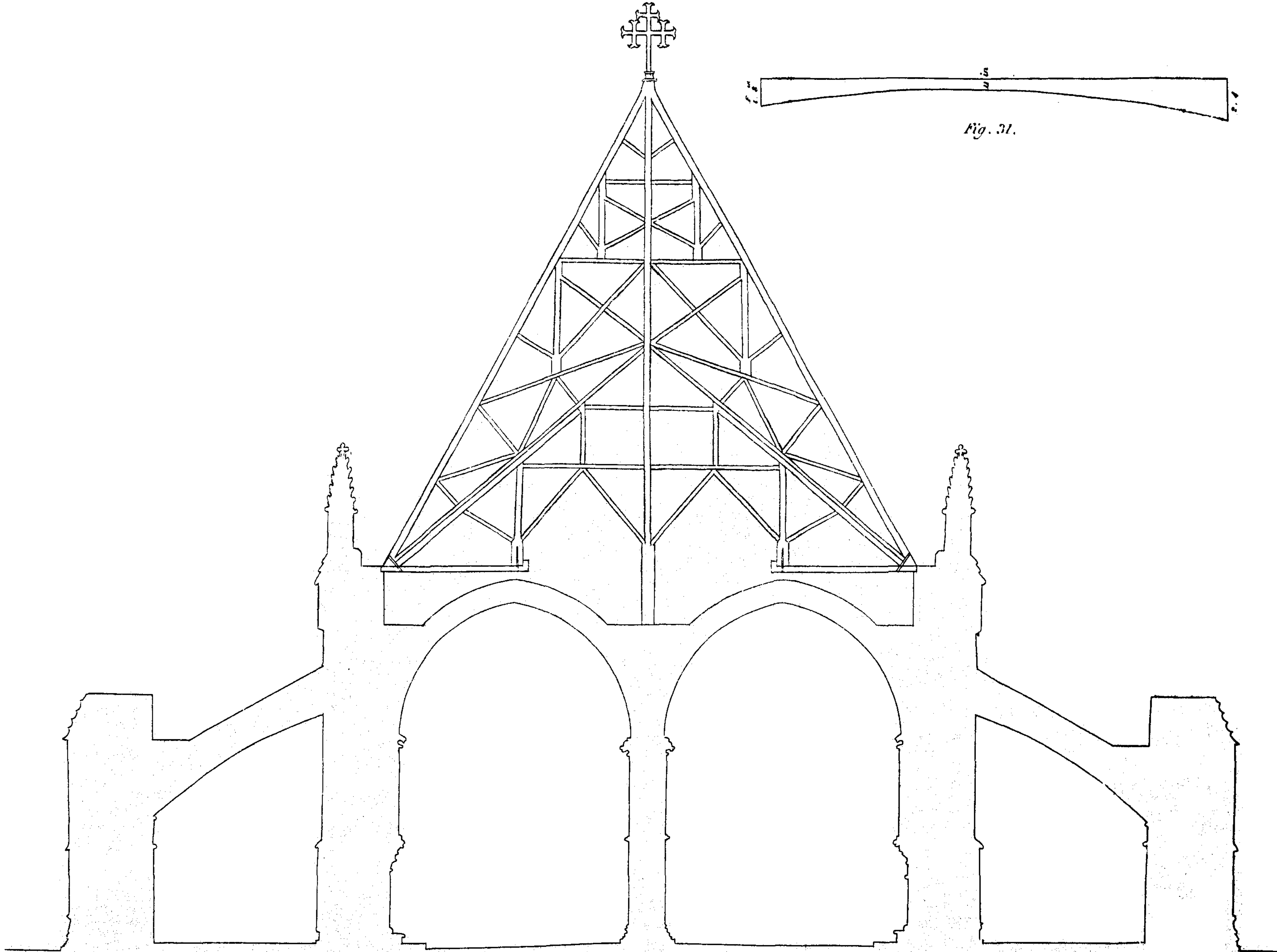
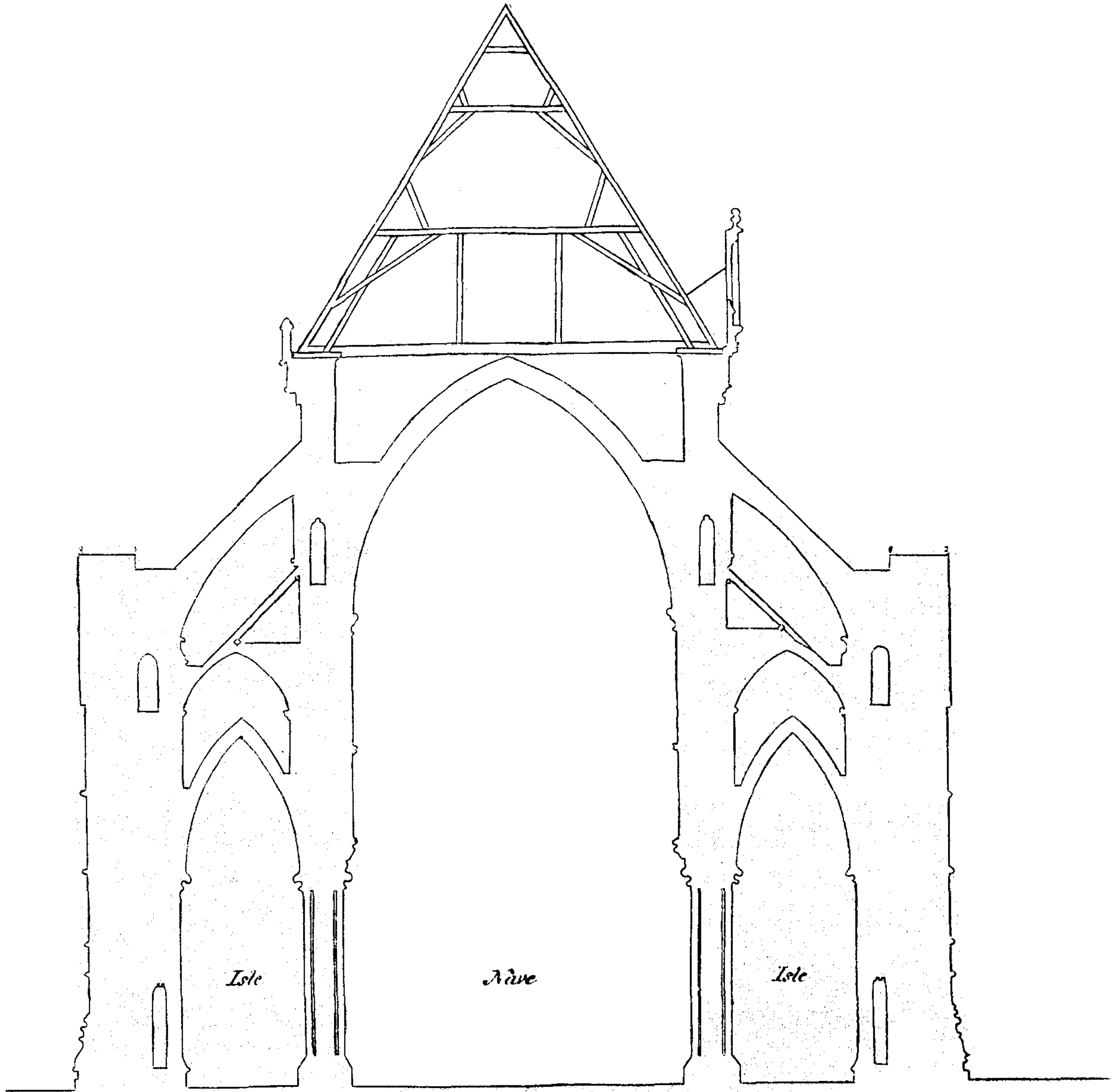
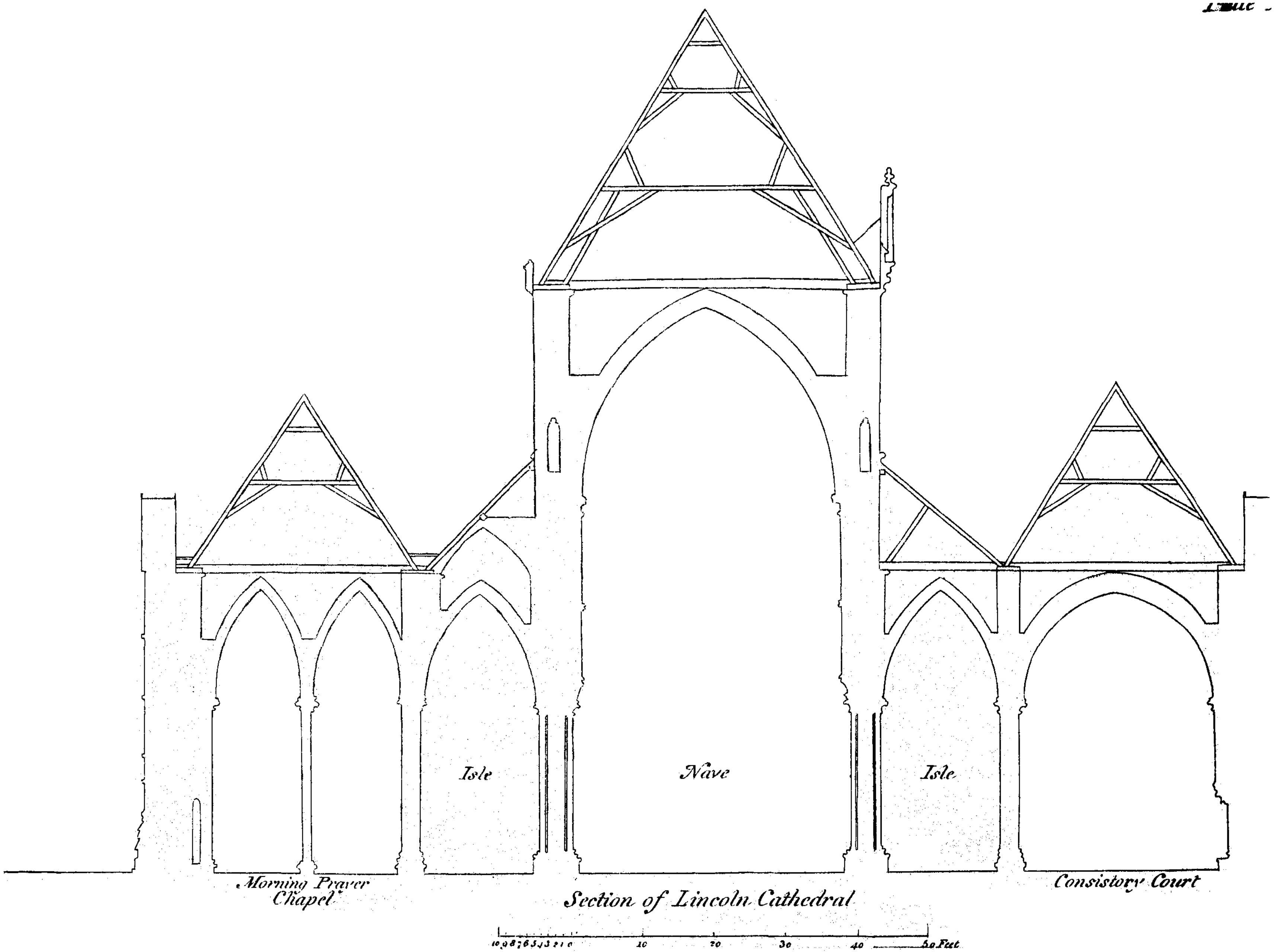


Fig. 31.

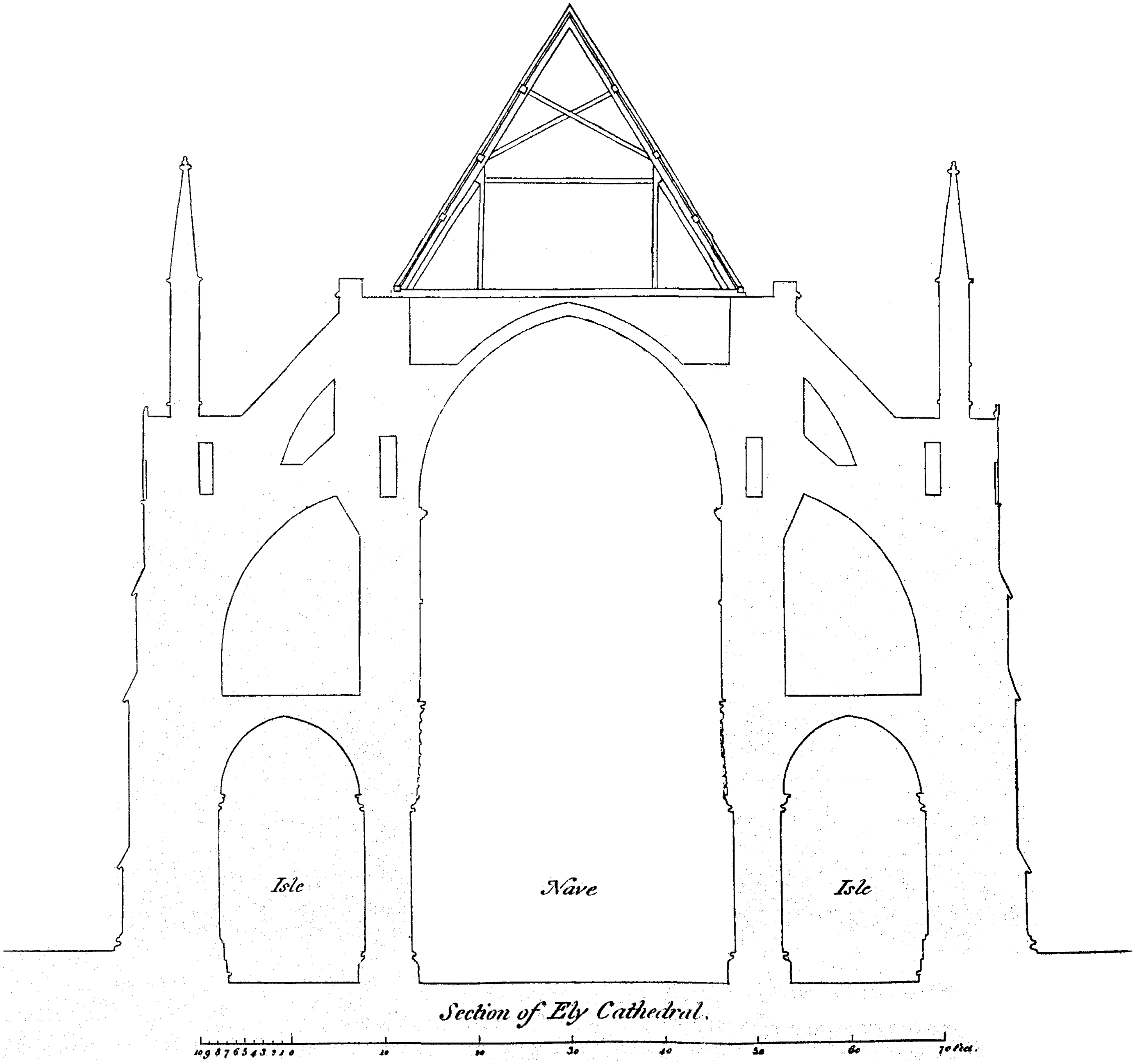
Section of the Chapter House Lincoln.



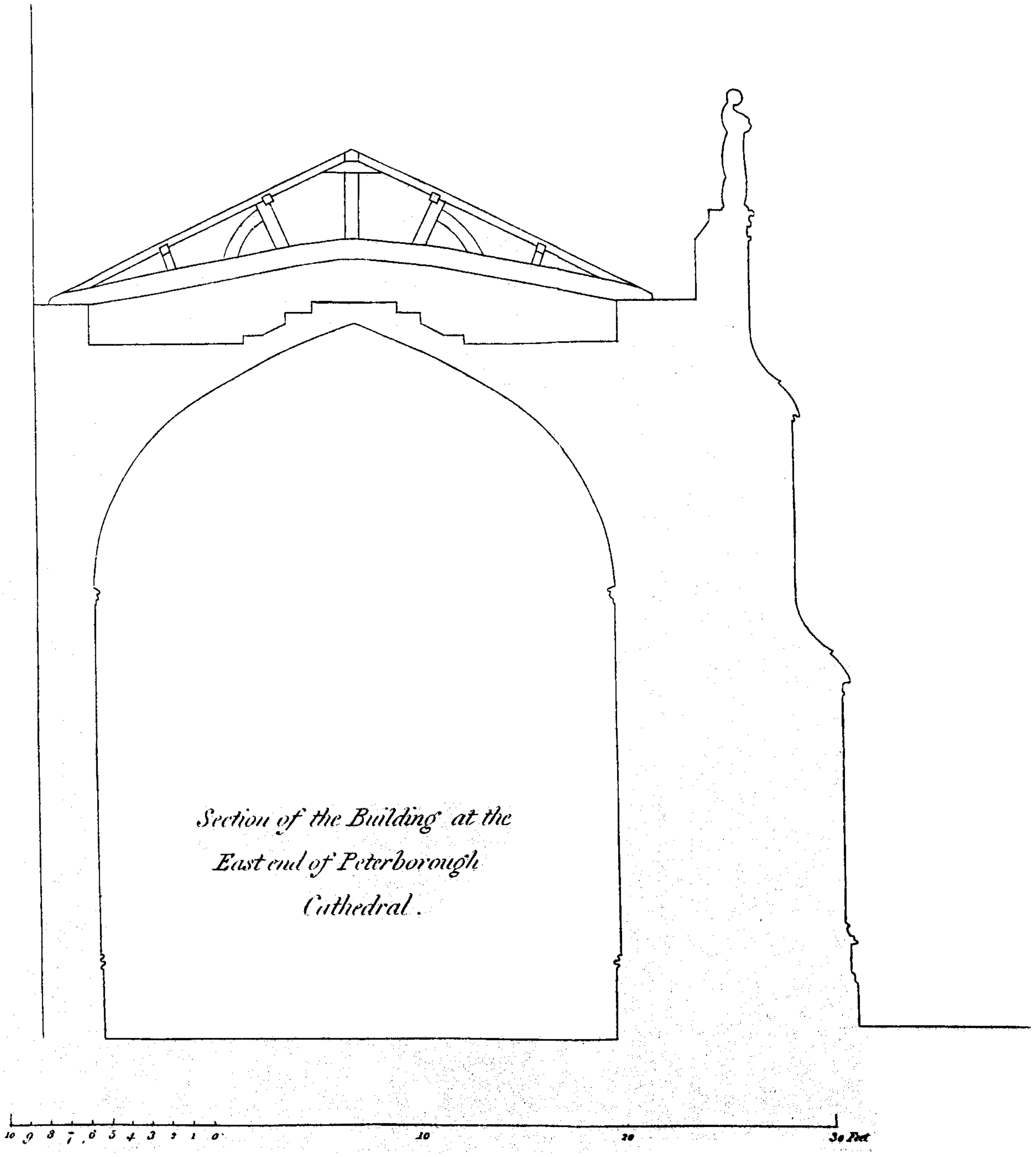
Section of Lincoln Cathedral



Section of Lincoln Cathedral

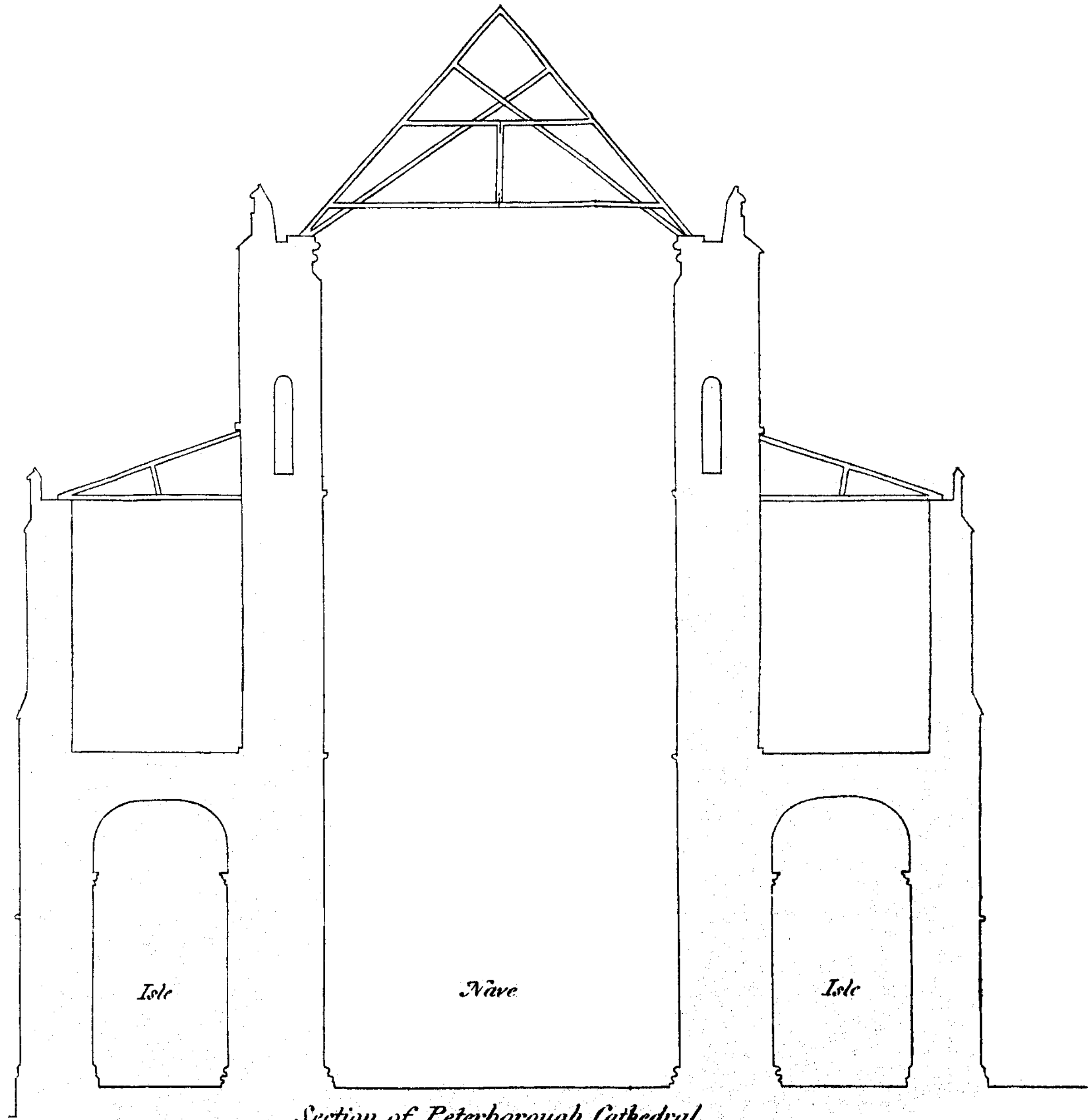


Section of Ely Cathedral.

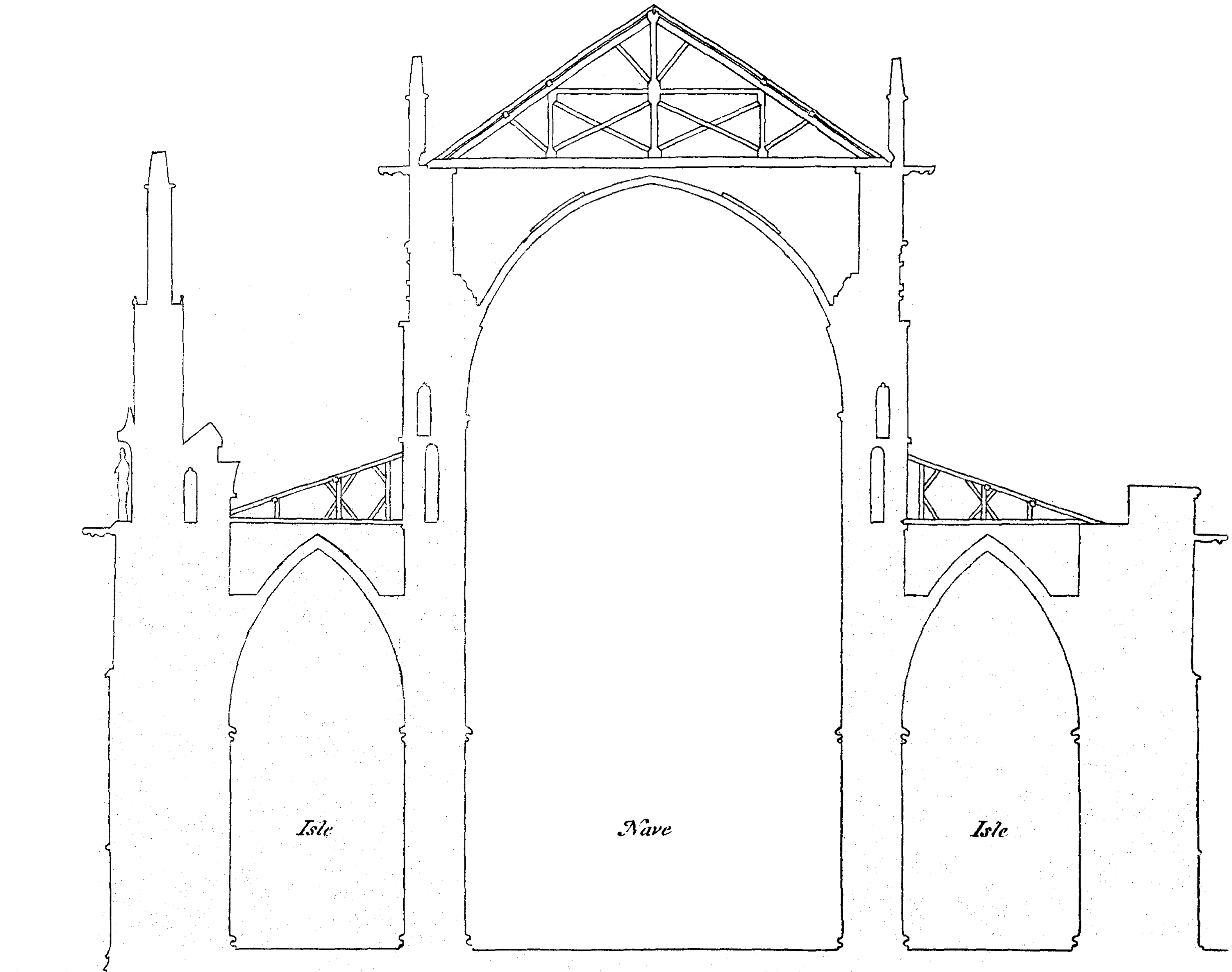


Section of the Building at the East end of Peterborough Cathedral.

Neele sculp. St



Section of Peterborough Cathedral



Section of York Cathedral.