Telford, Thomas y Alexander Nimmo. “Bridge”. En: *The Edinburgh Encyclopaedia*, ed. by D. Brewster. Edinburgh: W. Blackwood and J. Wangh, 1830. Vol. 4, pp. 479-545, plates 80-103.

Brick.

this substance, either baked or unbaked, float in water; and a twentieth part of argil may be added to their composition without taking away their property of swimming. These bricks resist water, unite perfectly with lime, are subject to no alteration from heat or cold, and the baked differ from the unbaked only in the sonorous quality which they have acquired from the fire. Their strength is little inferior to that of common bricks, but much greater in proportion to their weight; for M. Fabbroni found, that a floating brick, measuring 7 inches in length, 4½ in breadth, and one inch eight lines in thickness, weighed only 14¼ ounces; whereas, a common brick weighed 5 pounds 6¾ ounces. The use of these bricks may be very important in the construction of powder magazines and reverberating furnaces; as they are such bad conductors of heat, that one end may be made red hot, while the other is held in the hand. They may also be employed for buildings that require to be light; such as cooking places in ships, and floating batteries, the parapets of which would be proof against red hot bullets. The turrets which were raised on the ships of the ancients, says M. Fabbroni, were perhaps formed of these bricks; and perhaps they were employed in the celebrated ship, sent by Hiero to Ptolemy, which carried so many buildings, consisting of porticoes, baths, halls, &c. arranged in mosaic, and ornamented with agates and jasper.

Bricks appear to be of the highest antiquity; and, as we learn from sacred history, the making of them was one of the oppressions to which the children of Israel were subjected during their servitude in Egypt. The bricks of the ancients, however, so far differed from ours, that they were mixed with chopped straw in order to bind the clay together, and instead of being burned were commonly dried in the sun. Vitruvius recommended, that they should be exposed in the air for two years before they were used, as they could not be sufficiently dry in less time; and by the laws of Utica, no bricks were allowed to be used, unless they had lain to dry for five years. From Dr Pocock's description of a pyramid in Egypt, constructed of unburnt bricks, it appears that the Egyptian bricks were nearly of the same shape as our common bricks, but rather larger. Some of those he measured were 13½ inches long, 6½ broad, and 4 inches thick; and others 15 inches long, 7 broad, and 4¾ thick. The bricks used by the Romans were in general square; and M. Quatremere de Quincy observes, that in his researches among the antique buildings of Rome, he found them of three different sizes. The least were 7½ inches square, and 1½ thick; others 16½ inches square, and from 18 to 20 lines in thickness; and the larger ones 22 inches square, by 21 or 22 lines thick. Among the celebrated buildings of antiquity constructed of brick, were the tower of Babel, and the famous walls of Babylon, reckoned by the Greeks among the wonders of the world; the walls of Athens, the house of Crœsus at Sardis, and the walls of the tomb of Mausolus. The paintings, which were brought from Lacedæmon to Rome, to ornament the Comitium in the edileship of Varro and Murena, were cut from walls of brick; and the Temple of Peace, the Pantheon, and all the Thermæ, were composed of this material. The Babylonian bricks, which are in the possession of the East India company, and upon which Dr Hayes has lately favoured the public with a dissertation, are inscribed with various figures and characters, and are supposed by some to be a part of that brick work upon which Pliny tells us, that the Babylonians wrote the observations which they made of the stars for seven hundred and twenty years. See Fourcroy et Gallon, *Art du Tuilier Briquetier;* Jars *on making Bricks and Tiles;* Rozier *Introduction aux Observations sur la Physique, sur l'Histoire Naturelle, et sur les Arts;* Bergman's *Essays;* Nicholson's *Journal*, vol. ii. p. 498.; *Repertory of Arts*, vol. iii. p. 84.; and *Encyclopedie Methodique.* (L.)

BRIDE. See MARRIAGE.

BRIDEGROOM. See MARRIAGE.

BRIDGE.

Bridge.

THERE are few operations of art in which mankind are more deeply interested than in what relates to bridges. The ingenuity and hazard involved in constructing them; the numerous advantages derived from them; their being from objects of utility, in many instances, raised into all the magnificence which science and power can exalt them; justify us in treating the subject at considerable length, and endeavouring to exhibit under one article, most of the material circumstances which are connected with it.

In order to accomplish this, we shall first, in a cursory manner, trace the history of bridges; secondly, state the theory and principles upon which the rules, which ought to guide the engineer, are founded; and thirdly, explain what relates to the practice of bridge building.

HISTORY OF BRIDGES.

Bridge. History.

THE construction of perfect bridges, being a very complex operation, cannot have taken place amongst a rude and uninformed people; and in the course of this discussion it will be seen, that this did not always correspond with the progress of the other arts, even in situations where the intercourse was great.

The most obvious and simple bridge is that formed by single trees thrown across small streams, or, in case of broader streams, by fastening the roots of a tree on each bank, and twisting together their branches in the middle of the stream. These must have frequently occurred by chance, and they fall within the comprehension of the hunter; accordingly Mr Park found even the latter mode practised on rivers in the interior of Africa.

The next step is not much more complex, for in a space too great for the beforementioned operations, few manual arts were required to form ropes of rushes or leathern thongs, to stretch as many of them as were necessary between trees or posts on the opposite banks, and connect and cover them so as to form a slight bridge. The following accounts given by Don Antonio de Ulloa, will afford a distinct notion how these sort of bridges were constructed and used in the mountainous parts of South America. See vol. ii. page 164. London, 4th edit. 8vo.

"Over the river Desaguadero is still remaining the bridge of rushes, invented by Capac Yupanqui, the fifth Ynca, for transporting his army to the other side, in order to conquer the provinces of Collasuyo. The Desaguadero is here between eighty and a hundred yards in breadth, flowing with a very impetuous current, under a smooth, and as it were, a sleeping surface. The Ynca, to overcome this difficulty, ordered, four very large cables to be made of a kind of grass which covers the lofty heaths and mountains of that country, and called by the Indians, Ichu; and these cables were the foundation of the whole structure. Two of these being laid across the water, fascines of dry juncia and tortora, species of rushes, were fastened together, and laid across them. On these the two other cables were laid, and again covered with the other fascines securely fastened, but smaller than the first, and arranged in such a manner as to form a level surface; and by this means he procured a safe passage to his army. This bridge, which is about five yards in breadth, and one and a half above the surface of the water, is carefully repaired, or rebuilt every six months, by the neighbouring provinces, in pursuance of a law made by that Ynca, and since often confirmed by the kings of Spain, on account of its prodigious use; it being the channel of intercourse between those provinces separated by the Desaguadero."

Again, in vol. i. page 430: "When the rivers are too deep to be forded, bridges are made at the most frequented places. Of these there are two kinds besides those made of stone, which are very few: the former of wood, which are most common; and the latter of bujucos. With regard to the first, they choose a place where the river is very narrow, and has on each side high rocks. They consist of only four long beams laid close together over the precipice, and form a path about a yard and a half in breadth, being just sufficient for a man to pass over on horseback; and custom has rendered these bridges so natural to them, that they pass them without any apprehension. The second, or those formed of bujucos, are only used where the breadth of the river will not admit of any beams to be laid across. In the construction of these, several bujucos are twisted together, so as to form a kind of large cable of the length required. Six of these are carried from one side of the river to the other, two of which are considerably higher than the other four. On the latter are laid sticks in a transverse direction, and over these branches of trees as a flooring; the former are fastened to the four which form the bridge, and by that means serve as rails for the security of the passenger, who would otherwise be in no small danger from the continual oscillation. The bujuco bridges in this country are only for men, the mules swim over the rivers; in order to which, when their loading is taken off, they are drove into the water near half a league above the bridge, that they may reach the opposite shore near it, the rapidity of the stream carrying them so great a distance. In the mean time, the Indians carry over the loading on their shoulders. On some rivers of Peru there are bujuco bridges so large, that droves of loaded mules pass over them; particularly the river Apurimac, which is the thoroughfare of all the commerce carried on between Lima, Cusco, La Plata, and other parts to the southward.

"Some rivers, instead of a bujuco bridge, are passed by means of a tarabita; as is the case with regard to that of Alchipichi. This machine serves not only to carry over persons and loads, but also the beasts themselves; the rapidity of the stream, and the monstrous stones continually rolling along it, rendering it impracticable for them to swim over.

"The tartabita is only a single rope made of bujuco, or thongs of an ox's hide, and consisting of several strands, and about six or eight inches in thickness. This rope is extended from one side of the river to the other, and fastened on each bank to strong posts. On one side is a kind of wheel, or winch, to straighten or slacken the tarabita to the degree required. From the tarabita hangs a kind of leathern hammock, capable of holding a man; and is suspended by a clue at each end. A rope is also fastened to either clue, and extended to each side of the river, for drawing the hammock to the side intended. A push at its first setting off, sends it quickly to the other side.

"For carrying over the mules, two tarabitas are necessary, one for each side of the river, and the ropes are much thicker and slacker. On this rope is only one clue, which is of wood, and by which the beast is suspended, being secured with girths round the belly, neck, and legs. When this is performed, the creature is shoved off, and immediately landed on the opposite side. Such as are accustomed to be carried over in this manner, never make the least motion, and even come of themselves to have the girts fastened round them; but it is with great difficulty they are at first brought to suffer the girts to be put round their bodies, and when they find themselves suspended, kick and fling, during their short passage, in a most terrible manner. The river of Alchipichi may well excite terror in a young traveller, being between thirty and forty fathoms from shore to shore; and its perpendicular height, above the surface of the water, twenty-five fathoms."

A third mode of bridge building is, by constructing piers of stone at a distance to be reached by single stones or beams of timber; if used in shallow streams, and composed of rough stones, laid without mortar, it is likewise a very simple operation, and such as would readily occur to a very rude people; but if the stream was at all times deep and rapid, and the piers composed of hewn stone laid with or even without mortar, the case was very different; workmen must have previously been accustomed to quarrying, hewing, and transporting large stones, also building them in a regular mannner; working in metals, and

Bridge.

preparing mortar, must have been known; and, from what will be detailed under the head of Practice, it will be seen, that in preparing a proper foundation for each pier, the union and experience of various arts are required; and that the society, in which works of this sort, of any magnitude, were accomplished, was far advanced, and had the commmand of much well-regulated labour. The bridge over the Euphrates at Babylon, appears to have been constructed after this last manner; and there are many in different parts of China.*

With respect to the fourth mode, obtained by constructing arches of stone between the piers. If we may credit the accounts given by the Chinese, they constructed bridges in this manner, many centuries before arches were known to the inhabitants of the western world. Those connected with their inland navigation are numerous.

Chinese bridges.

From the accounts generally given, it is not easy to form distinct ideas of the dimensions or construction of the Chinese bridges, or to what extent they merit the appellations bestowed by travellers, of being great and magnificent. Duhalde informs us, that "the stone bridges are commonly built like ours, on large piers of stone capable of resisting the rapidity of the stream, and sustaining the weight of the arches, wide enough for the passage of large vessels. They are exceedingly numerous, and the Emperor spares no expence when the public good requires them to be built.

"Of these, there is one very remarkable at Foutchcou-fou, capital of Tou-kien. The river over which it is built is half a league in breadth; it is sometimes divided into small arms, and sometimes separated by small islands; these are united in joining the islands by bridges, which make altogether eight furlongs or Chinese lys and 76 toises. The principal of these has alone above one hundred arches built of white stone, with bannisters on each side handsomely carved, upon which, at the distance of every ten feet, are placed square pilasters, whose bases are very large, resembling hollow barks.

"But that which excels all the rest is at Suentchcou-fou, built over the point of an arm of the sea, without which the passage would be sometimes dangerous, even in a boat. It is 2500 Chinese feet in length and 20 in breadth; it is supported by 252 strong piers, 126 on each side. All the stones are of the same bigness, as well those which are laid from pier to pier, as those which are laid crosswise, insomuch that it is difficult to comprehend how stones of such an enormous size should be placed in that regular manner, or even raised on the high piers on which they lie. After this, there is nothing of the kind worth mentioning."

The only conclusion to be drawn from the foregoing description of this work, which excels all the rest, is, that two rows of large stones or piers, (each row consisting of 126,) have been set up across the shallow mouth of a river or arm of the sea; that, along the top of these, other long stones have been laid horizontally, like wooden beams; and lastly, that long stones have been laid crosswise upon those longitudinal beams, in the manner of joists in carpentry, or more probably close together, composing a compacted bed or roadway. By dividing 2500, the total length in Chinese feet, by 127, the number of openings, it gives nearly 20 feet between centre and centre of the piers, so that after the thickness of the pier is taken away from the 20 feet, a moderate opening is left for the lintel to cover. The dimensions crosswise, correspond with the description as to the stones being of the same size, (at least as to length); for the breadth of the bridge is said to be 20 feet, and taking away the thickness of the two longitudinal beams, leaves the dimensions of the opening to be covered by the stones lying crosswise. It is therefore to the carrying from the quarry and raising stones of this magnitude, that the praise of ingenuity must be attributed; there being nothing else in the mode of construction which has a claim to refined science, or great progress in the mechanical arts. The danger to boats passing, must, no doubt, have arisen from the shallowness of the water, and the frequency and violence of the surfs.

From the following relation, extracted from the same work, there is reason to expect correct information. It is entitled, "An account of the Journey of the Fathers Boures, Fontenay, Gorbillou Le Compte, and Vesdelore, from the port of Ning Po to Pekin, with a very exact and particular description of all the places through which they passed, in the provinces of Tche-kiang, Kiang-nan, Chan-tong, and Pe-tcheli.

"It is in this agreeable place that the city of Chao King has its situation. In the streets are a great number of canals, which give occasion for such a great number of bridges. They are very high, and have generally but one arch, which is so slightly built towards the top, that carriages never pass over them, which makes a great number of porters necessary. They pass over these bridges by a kind of stairs, of very easy ascent, and whose steps are not more than three inches in thickness. There are other sorts of bridges, made of stones 18 feet long, laid upon piles in the manner of planks. There are many of these over the great canal very handsomely built."

Again, "About four leagues from Hang-tcheou we crossed a village called Tan-si. It is built on both sides of the canal, on which are also two quays, about 400 or 500 geometrical paces in length. They are formed of the same freestone which lines the sides of the canal. There are stairs for the conveniency of every house, which are much better built, and more uniform than those in the city. In the midst of the village is a fine bridge of seven large arches; that in the middle is 45 French feet wide; the rest diminish in proportion to the descent of the bridge. There are two or three great bridges of one arch only.

"We crossed a great village or country town, called Ovan Kiang King, of large extent. One part communicates with the other by means of a bridge of three great arches, very curiously built: the middle arch is 45 French feet wide and 20 feet high."

Of these arches, which are here termed large and great, we find the span to be only 45 feet; an extent

* In Fig. 1. of Plate LXXX. we have represented the probable steps by which the arch was nvented.

Bridge.

which, in Europe, would not be honoured with those appellations.

We have also heard of a bridge over a river named Laffrany in China, which joins two mountains together, said to be of one arch 600 feet span, and 750 feet in height; but having no distinct authority for this, and its being so very unlike to those described in Duhalde's work, we mention it merely as a matter deserving of more enquiry.

But it is of real importance to notice the description given by Mr Barrow of the mode in which some of the arches in China are constructed. "Each stone from five to ten feet in length, is cut so as to form the segment of the arch, and in such cases there is no key stone; ribs of wood fitted to the convexity of the arch, are bolted through the stones by iron bars fixed into the solid part of the bridge; sometimes they are without wood, and the curved stones are mortised into long transverse blocks of stone."

In Egypt and India, from whence the western world derived the rudiments of many sciences and arts, the construction of the arch was totally unknown; for the magnificent temples of the latter, and the splendid tombs of the former, were produced by cutting matter away in the manner of sculpture. There is no trace of the arch met with in the ancient works of Persia or Phœnicia; and even the Greeks, who created a school of architecture and sculpture, and carried it to the utmost degree of perfection of which it was capable, have a very doubtful claim to the knowledge of the arch. It is certain they never used it as an external feature of their temples, much less in the construction of bridges over rivers; and it has been observed, that the great Pericles, while he adorned the city of Athens with splendid edifices, never constructed a stone bridge over the small river Cephisus, although upon the most frequented road to that city. It is therefore to the Romans that the western world is indebted for this singularly useful application of architecture.

There is no certainty respecting the time when the Romans first used arches: If the Cloacae of Rome were really constructed in the time of the elder Tarquin, the use of arches must have then been well known; and from that prince's origin and connection, it is probable that they would be the labours of Tuscan workmen. It has been positively said by some, that the Romans received their knowledge of the arch from the Tuscans, who were at that time much farther advanced in the arts than their Italian neighbours. If this is admitted, the first knowledge of the arch is at least very intimately connected with Greece, the Tuscans being acknowledged as a colony of Dorians.

Roman bridges.

Whatever doubtful circumstances attend the claim to the invention of the arch, we know, from the best historical evidence, that the Romans first applied it to works of general use, as in forming aqueducts for conveying water to large cities, constructing bridges over rivers, vaulting magnificent temples, and in erecting monuments for recording the actions of their greatest heroes.

We at present consider only their bridges. At or adjacent to Rome, Gautier mentions eight bridges.

1. Pons Ælius, built by the Emperor Adrian, and named after him. It is said to have once had a cover of bronze supported by 42 columns. It is now called Sancto Angelo.

2. A triumphal bridge, the ruins of which are now seen in the Tiber. The emperors and consuls passed over this bridge when they were decreed a triumph.

3. Pons Janiculensis, now Ponto Sixtus, it having been rebuilt by Pope Sixtus IV. in 1475.

4. Pons Cestius, at present St Bartholomew. It was rebuilt by the Emperor Valentinian.

5. Pons Fabricius, now Ponto Caspi.

6. Pons Senatorius, at present Sancta Maria.

7. Pons Horatius formerly Sublicius, built of stone by Horatius Cocles; rebuilt by Emilius Lepidus; the ruins are still seen in the Tiber.

8. Pons Milvius, which is about two miles out of Rome upon the Flaminian way.

Palladio gives a description of the bridge of Rimini, built by the Romans also upon the Flaminian way, which has five arches; likewise that of Vicenza upon the Bachliglione, of three arches; and of one upon the Rerone, of three arches.

And Martinellis mentions a bridge near Narni, on the road from Rome to Loretto, built by Augustus. It consisted of four arches, the first 75 feet span and 102 high: the spans of the others were 135, 114, and 142 feet. This appears to be the most magnificent bridge the Romans constructed in Italy.

In the provinces the Romans built many bridges, some very magnificent. We shall instance two in Spain, both in the province of Estremadura. That of Merida is upon the river Guadiana. Don Antonio Ponz, in his *Viage de Espanâ*, says he found its length 1300 paces; Vargas reckons 64 arches. In the time of Philip III. one of the large arches towards the middle was destroyed by an inundation, on which account three or four adjoining were rebuilt in 1610.

But perhaps the most magnificent of all the Roman bridges, and one of the noblest monuments of antiquity, is, the bridge of Alcantara upon the Tagus, at the town of that name. The town has probably taken its name from that structure, as the word *alcantara*, in the Arabic, signifies a bridge. It consists of six arches: its whole length is 670 Spanish feet, and from the bottom of the river to the roadway the height is 205 feet. For these Roman bridges, see Plate LXXXII. Plate LXXXII.

Besides these ancient bridges which still exist, or are correctly described, we have accounts of many others; as that of Darius upon the Bosphorus of Thrace, Xerxes upon the Hellespont, Pyrrhus upon the Adriatic Gulf, Cæsar upon the Rhine, and Trajan upon the Danube; but these were constructed for the temporary purposes of war. The descriptions are vague, some of them improbable, and they belong more to military than civil architecture.

The ancient aqueducts, which were magnificent, will be described under the head of Inland *Navigation*.

From the destruction of the Roman empire, to the establishment of modern Europe, it is in vain there to enquire for the progress, or expect the improvement of bridge building. In this, however, we ought to except the fine works of the Moors in Spain, particularly the bridge of Cordova over the Guadalquivir, built by Issim, the son and successor of Abduel Akman, the first of the Moorish kings of Spain.

Revival of bridge building in Europe.

When the arts began to revive in Europe, it was

Bridge.

chiefly towards religious structures that power and influence were directed. One singular instance occurs of enthusiasm being directed to the useful purposes of improving the passages over rivers. Gautier, upon the authority of Magna Agricola of Aix, says, that upon the decline of the second, and commencement of the third race of kings, the state fell into anarchy; and that there was no security for travellers, particularly in passing rivers, where violent exactions were made by banditti. To put a stop to these disorders, sundry persons formed themselves into fraternities, which became a religious order, under the title of Brothers of the Bridge. The object of this institution was to build bridges, establish ferry boats, and receive travellers in their hospitals on the shores of rivers. The first establishment was upon the Durance, at a dangerous place named Maupas; but in consequence of the accommodation arising from this establishment, the same place acquired the name of Bonpas. He relates further, that St Benezet, who proposed and directed the building of the bridge of Avignon, was a shepherd, and that he was not twelve years of age when repeated revelations from heaven commanded him to quit his flock and undertake this enterprize; that he arrived at Avignon just at the time the bishop was preaching to fortify the minds of the people against an eclipse of the sun, which was to happen the same day. Benezet raised his voice in the church, and said he was come to build a bridge. His proposition was accepted by the people with applause, but rejected with contempt by the magistrates, and by those who thought themselves wisest. As it was at that time an act of piety to build bridges, and Avignon being then a popular republic, the people prevailed, and every one contributed to the good work, some by money, and some by labour, all under the direction of Benezet, aided by the brothers. And he, by performing a great number of miracles, animated the zeal of every body. Upon the third pier was erected a chapel to St Nicholas, protector of those who navigate rivers. This was done after the death of Benezet, which happened in 1184. His tomb became celebrated for pilgrimages, where many miracles were performed. He had taken care to establish a conventual house, and a hospital, leaving the brothers to continue the work of the bridge.

Bridge of Avignon.

This bridge, which was composed of 18 arches, was begun in 1176, and completed in 1188. In 1385, during the contentions of the Popes, some of its arches were destroyed; three others fell in 1602, from the neglect of repairing a fallen arch. In 1670, the frost was so great, that the Rhone for several weeks bore the heaviest carriages: when the thaw followed, the ice destroyed the piers; but the third pier, with the chapel of St Nicholas, has stood notwithstanding all these accidents.

Our admirable bridge saint, not the least useful of that once numerous class of enthusiasts, in accomplishing, under such circumstances, so difficult, valuable, and magnificent a work as the bridge of Avignon, has perhaps quite as just a claim to the power of performing miracles as most of the saints of his day; and it is not improbable, that from the influence of his tomb, and that of his mantle upon the surviving brethren, that the still greater bridges of Lyons of 20 arches, and St Esprit of 19, were accomplished. Parts of the elevation of those bridges are given in Plate LXXXIII.

Bridge.

Plate LXXXIII.

Perronet states, that in 1354 an arch of 150 French feet, or 160 English feet span, was built at Verona; and in 1454, one of 172 French feet, or 183.8 English span, and 66 French feet, or 70.6 English of rise from the springing, at Vielle-Brioude upon the river Allier in France. The last mentioned is the greatest span we know of for a stone arch.

In Italy there are many fine bridges. The peculiar situation of Venice has required a number far beyond what is to be found in any other city. The finest is the Rialto of 98½ feet span, and 23 feet rise. It was designed by the celebrated Michael Angelo, and erected between 1588 and 1591. Gautier states, that the numbers in the different quarters of the city are as follows, viz.

Bridges in Italy.

In the quarter of St Paul . . .	37
La Croix . .	35
Canal Regio .	75
Arsenal . . .	72
Isle du Juifs .	9
Derso Duro .	67
St Marc . . .	44
	339

In France, during the two last centuries, many fine bridges have been erected. The Pont Royal over the Seine at Paris, in 1685, from a design of Mansard. It consists of five arches. The centre one 77 French, or 82.3 English feet span; the breadth, including parapets, 56 French, or 59.9 English feet. The bridge at Blois, built from a design of the Sieur Gabriel, consists of 11 arches; the centre one 86 French, or 91.10 English feet span; the breadth over the parapets, 50 French, or 53.5 English feet.

Bridges in France.

Perronet gives plans and descriptions of sundry large bridges in France, constructed between the years 1750 and 1772.

The first in point of time and magnitude is the new bridge upon the Loire, at Orleans. The ancient bridge, which consisted of 19 arches, was in so bad a state, that it was found necessary to construct a new one; for which a design, made by M. Hupeau, then first engineer of bridges, was preferred; and by that time such was the progress made in bridge building, that, instead of 19 arches, as in the old edifice, this new design consisted of only nine. The middle arch is 100 French, or 106.9 English feet span, and the rise or versed sine 28 French, or 29.11 English feet. The arches next the abutments are 92 French, or 98.3 English feet span, and rise 25 French, or 26.8 English feet: the others are in proportion. The breadth, including the parapets, 46 French, or 49.2 English feet. It was begun in 1750, and opened to the public in 1760. Nothing can exceed the simplicity and elegance of this bridge, as will appear by the elevation, one half of which is given in Plate LXXXIV.

Bridge at Orleans.

Plate LXXXIV.

Of the same beautiful simplicity of character is the design of the bridge upon the river Seine, at Mantes. It consists of three arches: the middle arch is 120 feet French, or 128.2 English feet span, and rise 36 French or 38.5 English feet; the two side arches

Bridge at Mantes.

Bridge.

are 108 French, or 115.4 English feet span, and rise 32 feet 6 inches French, or 34.9 English; the breadth, including the parapets, is 33 French, or 35.3 English feet. It was also designed by M. Hupeau; begun under his direction in 1757; and during that and 1758, raised to the level of the sixth course of arch stones: the work was then suspended on account of a war. It was recommenced in 1763 under the direction of Perronet, (M. Hupeau having died in that interval,) and was opened in 1765.

Bridge at Neuilly.

One of the finest of the French bridges, and the greatest work of Perronet, is that upon the Seine at Neuilly. It consists of five arches, each 120 French, or 128.2 English feet span, and 30 French, or 32 English feet rise; the breadth, including the parapets, is 45 French, or 48 English feet. It was begun in April 1768, and opened in October 1773; the masonry was completed in 1774; the roads, and other operations connected with this bridge, were finished in 1780. A great peculiarity in this bridge, well deserving the attention of engineers employed in similar works, is, that the soffits of the arches are shaped to suit the contracted vein of water, as formed in the entrance and exit of pipes. This is accomplished, by making the general form of the body of the arch elliptical, with a rise of 30 French, or 32 English feet, or $\frac{1}{4}$ of the span; but making the headers follow the segment of a circle, the versed sine of which is only $13\frac{1}{2}$ French, or 14.5 English feet, or about $\frac{1}{9}$ of the span. This, besides affording facility for the passage of flood waters, gives a great appearance of lightness to the whole fabric. The effect will be seen in the elevation, one half of which is given in Plate LXXXIV.

Plate LXXXIV.

In the bridges at Orleans, Mantes, and Neuilly, we find the rise of the arches to be between $\frac{1}{3}$ and $\frac{1}{4}$ of the span; but rendered confident by success, and desirous of giving a variety to his works, Perronet, in the bridge over the Oise, at St Maxence, consisting of three arches, each 72 French, or 76.10 English feet span, makes the rise only 6 French, or 6.5 English feet, or $\frac{1}{12}$ part of the span; also, instead of making the piers each a solid, right lined on the sides, he divides each pier into two distinct parts, with an open space between them, and composes each part of two columns united by a piece of plain wall. The deviation from former works of a similar kind is here certainly sufficiently distinct; but the propriety of this measure will be discussed under the head of Practice. This bridge was begun in 1774; the operations were suspended in 1775; they were recommenced in 1780; and the centers were struck in 1785. See Plate LXXXV.

Bridge at Maxence.

Plate LXXXV.

Bridge at Saumur.

The bridge built upon one of the arms of the Loire at Saumur, from a design of M. de Voglio, and executed chiefly under the inspection of L. A. de Cessart, may also be classed amongst the first of the French bridges. It consists of 12 elliptical arches, each 60 feet span, and 21 of rise; the piers are 12 feet thick, and the breadth of the bridge, including the parapets, is 42 French, or 44.9 English feet. It was begun in 1756, and finished in 1770.

Bridges in England.

The progress of bridge-building in England seems to have kept pace with the same art on the continent. The very singular bridge at Croyland in Lincolnshire, is said to have been built in 860. This date is likely to be correct; for Croyland Abbey was founded in 716, and the Abbey of Rumsey, in Huntingdonshire, in 974. (Bentham's *Essays*.) This bridge has three distinct approaches, formed by three segments of a circle, which meeting in the middle, compose pointed arches, their bases or abutments standing upon the points of an equilateral triangle.

Croyland bridge.

It is worthy of remark, that the bridge of Avignon was begun under the direction of Saint Benezet in 1176, and that of London begun to be built of stone under the direction of Peter of Colchester, a priest, in the same year (1176.) The French "Brothers of the Bridge" accomplished their magnificent and useful work in 12 years, the labours of the English priest occupied 33 years; but this may be accounted for, by considering the interruptions which must be experienced in a river, where the tide rises twice every day from 13 to 18 feet. We may further remark, that as the constructions of the bridges of St Esprit and Lyons immediately succeeded to that of Avignon, so the bridge at Newcastle-upon-Tyne was built of stone in 1281, and that over the Medway at Rochester, consisting of 11 arches, much about the same time.

London bridge.

In London bridge there are now 19 arches, and it is 45 feet in breadth. For many ages there were houses along each side of it; but these were removed, the middle pier was taken away, and the space, including the two adjacent arches, converted into one arch of 72 feet span, in 1758. The remaining old arches are very narrow, and the piers enormously large, being from 15 to 25 feet in thickness above the sterlings. The passage over the bridge is very commodious, but in other respects it is very inferior to the before mentioned old French bridges. See Plate LXXXIII.

Plate LXXXIII.

Many other old English bridges might be described, which, in conformity with the turbulence of the times, were generally fortified with gateways. It would be curious to trace their history, and delineate their features; but as the limits of our present article will not admit of this, we shall pass on to those of modern times, from which our readers will derive more useful information.

In 1636, the English Palladio (Inigo Jones) gave a design for a bridge, which was erected at Llanwst in Denbighshire. It consists of 3 arches, segments of circles; the middle one is 58 feet span, and rises 17 feet, the piers are 10 feet thick, and the breadth of the soffit of the middle arch is 14 feet. The arch stones of the largest arch being only 18 inches deep, the covering over them being little, and the approaches very steep, the bridge has a very light appearance.

Llanwst bridge.

The bridge over the river Thames at Westminster, being not only the greatest work of the kind in England, but having, in what regards laying foundations in deep water, and constructing centers for large arches upon navigable rivers, formed a new school for bridge-building in this island, we shall give a detailed account of it.

Westminster bridge.

From reports made by Mr Labalye, (1751) it appears, that from 1734 to 1738, the time was employed in obtaining acts of parliament, and determining the precise situation and plan of this great work.

Bridge.

Westminster bridge.

In 1738, the situation was finally determined to be a little way below New Palace Yard; the model made by M. Labalye was approved of, and he was appointed engineer. The intention at this time was to construct the piers of stone, and place a wooden superstructure upon them. This latter part was designed by Mr James King, who contracted to complete it in 12 months after the piers were finished for £28,000.

It was not till after many explanations and discussions, that Mr Labalye satisfied the commissioners, appointed by parliament, of the facility, economy, and security to be derived from laying the foundations of the piers in caissons or chests, instead of placing them upon piles in the ancient manner, cut off about the level of low water; or using batterdeaux or coffredams, formed around the foundations, and pumping the water from the inside, as had been performed in more modern times. This beautifully simple mode was, however, adopted, and the first stone of this great fabric was laid by the Earl of Pembroke on the 29th January 1739. During the same year, the commissioners directed Mr Labalye to prepare a design for a superstructure of stone, which he did, and it was approved of and adopted on the 31st January 1740: A liberal arrangement having been made with Mr King respecting his contract for the wooden superstructure, he immediately designed, for the stone arches, those excellent centres, which have ever since served as a model for works of a similar kind in England.

The works were carried on with great dispatch and success; the centre of the last arch was struck on the 25th July 1747, and, on the 14th November, the roads and streets were finished. A circumstance, however, took place, which prevented the bridge from being, at that time, opened to the public. The workmen employed to get gravel out of the bed of the river to cover the roadway of the bridge, finding some very suitable near the third pier, on the western side of the centre arch, they excavated considerably lower than the foundation, and too near it; the gravel then run from under the platform, and the pier sunk so much as to render it necessary to take down the two arches which rested upon it. The securing the foundation, rebuilding the pier and two arches, and replacing the parapets, pavements, and roadway, was completed, and the bridge opened to the public on the 18th Nov. 1750. This bridge consists of 13 large and two small arches; their forms are semicircular; the middle one is 76 feet span, and the breadth over the parapets 44 feet. See Plate LXXXVI.

Plate LXXXVI.

Blackfriars bridge.

About 10 years after the completion of Westminster bridge, another was begun to be erected between it and London bridge, now well known by the name of Blackfriars. The design was made by Robert Mylne. It consists of nine arches of an elliptical form; the middle one is 100 feet span, and the breadth across the bridge is 43 feet 6 inches. Mr Mylne benefited by the example of Labalye, and built the piers in caissons; but probably alarmed by the sinking of one of the piers at Westminster, he drove piles in the spaces upon which the bottoms of the caissons are placed. His arches being of wider span and of an elliptical form, his piers of proportionally less thickness, and having less masonry over the top of the arches, this bridge has a much lighter appearance than that of Westminster. It has been doubted, whether the slender detached Ionic columns are a proper accompaniment to such a work, and whether the divisions of the lengths of the rusticated headers of the arches are any improvement. His centers are evidently a copy of those used at Westminster.

The general style of this bridge bespeaks a mind emboldened by the success of his predecessor, to advance, though very cautiously, a step further in the practice of bridge-building. It is a work of great merit, and will not suffer by a comparison with any other constructed in the same age. It was begun in 1760, and completed in 10¼ years. See Plate LXXXVI.

Plate LXXXVI.

Bridge at Perth.

At the same time that this noble work was carrying on at Blackfriars, a very fine bridge was constructing upon the river Tay, at the town of Perth in Scotland. It consists of nine arches; the middle one is 77 feet span, the width across is 26 feet, and the total length is 906 feet. It was designed and executed under the direction of Mr Smeaton, between 1760 and 1771. About the same time, and under the direction of the same gentleman, a bridge of no inconsiderable magnitude was built over the river Tweed at Coldstream, consisting of 5 arches, the middle one being 64 feet span; and also a bridge of seven arches over the river North Esk, near Montrose.

Bridge over the Esk.

North bridge at Edinburgh.

Previous to forming the plan of that magnificent extension of the city of Edinburgh, known by the name of the New Town, it was necessary to form a commodious communication with the central part of the High Street of the Old Town. This was accomplished by constructing a bridge over the deep valley called the North Loch. This bridge consists of three arches, each about 72 feet span, and two small arches each 20 feet span; the height from the present surface of the ground to the springing of the arches on the piers, is 17 ft. 6 in.; the arches being semicircular, rise 36 feet, the archstones are 2 ft. 9 in., and from the top of the archstones to the top of the parapets, is 9 ft. 9 in., making the whole height, from the surface of the ground to the top of the parapet over the middle arch, 65 feet; the breadth across the soffit of the arches is 42 ft. 3 in. From the arches to the banks on each side of the valley, the spaces are occupied by coach houses, stables, &c. formed under the roadway. The outline of the cornice and parapet, contrary to usual practice, is a curved line, bending downwards. This, in viewing the elevation, gives the appearance of the middle part of the bridge having sunk. Excepting the arches, which are fine and well executed, no part of the design claims much commendation; and we cannot help regretting, that the architect Mr Mylne, has failed in rendering this structure a suitable feature to the singularly fine situation which it occupies.

Bridges over the Thames & the Severn.

Several excellent stone bridges have lately been constructed upon the river Thames at Kew, Maidenhead, Henley, and Oxford; and not inferior to these are the bridges over the river Severn in Worcestershire and Shropshire. Five of these were designed and executed under the direction of Mr Gwynn, a native of Shrewsbury, and two by our countryman Mr Telford, the engineer.

Bridge.

The centre arch of the bridge in Blenheim Park, is 101 ft. 6 in. span.

In 1762, a single arch was built upon the river Tees, at Winstone in Yorkshire, from a design of Sir Thomas Robinson, the span of which is 108 feet 9 inches.

Bridge over the Ouse.

A very fine bridge has been lately erected at Ferry Bridge, in the same county, upon the river Ouse: it consists of three arches, the centre one 65 feet span, the width within the parapets 28 feet 6 inches.

Bridge over the Taaf.

In South Wales, the bridge over the river Taaf, near Llantrissart, in Glamorganshire, is justly celebrated, both on account of its great span, and the singular circumstances which attended its construction. In 1746, William Edwards, a country mason, undertook to build a bridge at that place. He built one of three arches, and it was allowed to be well executed; but, being in a mountainous district, the torrents sometimes rise hastily to a height which must appear incredible to the inhabitants of flat countries. One instance of this sort happened after this bridge had stood about two years and a half, when the torrent carried along with it trees, brushwood, hay, and whatever of this kind lay in its way: these were entangled in the arches, and, causing the water suddenly to create a great head, the bridge was swept away. As William Edwards had given security to maintain the bridge for seven years, he immediately set about rebuilding it; but, in order to avoid future injuries from similar torrents, he constructed one arch, the segment of a circle of which the cord line was 140 feet, and the versed sine 35 feet. The arch was finished, but had not received the parapets, when the weight pressed in the haunches, raised up the crown, and destroyed the arch. This was in 1751. But William Edwards, possessed of an uncommon degree of fortitude, resolved to rebuild the arch of the same dimensions; and it appears he took his measures prudently; for we have been informed, by that excellent and respectable engineer Mr Jessop, who was then a clerk with Mr Smeaton, that Mr Edwards consulted that eminent man with respect to rebuilding this large arch. Mr Jessop does not recollect the advice which Mr Smeaton gave, but the bridge was rebuilt in 1755; the chord line and versed sine are the same as before, and the width across is eleven feet: in each haunch or spandrel there are three cylindrical arches quite across the bridge; the lowest is nine feet, the middle six feet, and the uppermost three feet diameter. These, to avoid weight, are of course left hollow; and Mr Evans, a native of Wales, who was afterwards chief engineer to the Royal Canal in Ireland, informed Mr Jessop, that, in addition to the cylinders being left hollow, the spaces between them were filled up with charcoal. From the steepness of the ascent on each side, and the narrowness across, this bridge is more remarkable as an effort of art than for the accommodation it affords.

Bridges in Scotland.

Of late years, the building of bridges has been carried to a very great extent in Scotland. Upon the river Teviot, immediately above its junction with the Tweed, a very handsome stone bridge of three arches has been constructed from a design, and under the direction of Mr Elliot, an architect resident in Kelso. The middle arch is 65 feet span, and rises 17 feet; the arches are segments of circles, and the width over the parapets is 23 feet. There are coupled columns over the piers, which are quite insulated; and the points of the piers are in the shape of Gothic arches. It was begun in 1794, and finished in 1795. About 4 years after the completion of this bridge, another, very elegant one was constructed at Kelso, from a design of Mr Rennie, an eminent engineer. It is situated immediately below the confluence of the Teviot with the Tweed. It consists of 5 arches, each of 73 feet span, and 21 feet rise; they are of an elliptical form, and the road over them is level. Over each pier, and upon each abutment, are two small columns, and an entablature runs along the whole of the bridge. The columns are not insulated, being $\frac{3}{4}$ columns only; the points of the piers are semicircular; the width over the parapets 26 feet. It was begun in 1799, and finished in 1803. The characters of those two bridges being rather gentle than bold, accord well with the beautiful scenery of the adjacent banks of those two fine rivers.

Bridge over the Teviot.

Bridge at Kelso.

On the road from Berwick to Edinburgh, the Peas bridge erected over a deep dingle, is a bold work. It consists of four arches; the largest span is 55 feet, and the height of the bridge is 124 feet. The architect was Mr David Henderson of Edinburgh.

Peas bridge

A large arch has been built at Aberdeen also over a dingle, through which there runs a small rill called the Den Burn; it forms a part of an improved approach to the city from the southwards. The magistrates had, in the year 1801, begun to construct a bridge of three small arches, and had laid the foundations of the abutments and piers for that purpose, under the direction of their then superintendant of city works, Mr Fletcher, when Mr Telford the engineer passing that way on the service of government, was desired by the magistrates to examine their intended bridge. On considering the excellent granite stone which was used, he prevailed with them to abandon the scheme of having three arches. At their desire he gave a plan of one arch of 150 feet span, being larger than any stone arch in Britain, and otherwise containing many singular features calculated to prove what could be performed with Aberdeen granite. But however desirous the magistrates were to exhibit the excellency of their favourite material, the expense of this plan much exceeded their funds. Mr Telford afterwards made a simpler design; but in order to save some masonry of the abutments which had already been executed, they got their inspector of the city works to reduce the span to 130 feet, of which dimensions it has been executed. The rise is 29 feet, and breadth across the soffit 43 feet. It is still a magnificent arch, though of smaller span than that of Mr Edwards over the Taaf. The difficulty attending the construction of a large arch here, was much lessened by its being placed on dry land.

Bridge at Aberdeen.

A much more arduous task has been accomplished upon the river Dee, at Tongueland, near Kirkcudbright, where there is about 10 feet of water in the lowest state of the river, above which the ordinary spring tides rise 16 feet, and where, of course for a large arch, a trussed center was required. The design was given by Mr Telford; it is 118 feet span, and the rise or versed sine 38 feet. The spaces be-

Bridge at Tongueland.

Bridge.

tween the large arch and the rocky banks upon which it abuts, instead of being filled by earthen embankments, are occupied by small arches raised upon slender piers. The whole has a bold effect, especially during the flux and reflux of a high spring tide immediately under the bridge, when agitated by a strong westerly wind, and accompanied by a great land flood, tumbling down a rocky channel at some distance above it. See Plate LXXXVII.

Plate LXXXVII.

Bridge at Fochabers.

A large bridge has been lately built over the river Spey, near Gordon castle, at Fochabers. It consists of four arches; the two middle ones are each 95 feet span, and the breadth over the parapets is 21 feet 6 inches. There being an even number of arches, a pier is brought into the middle of the river, and the architecture of the facade is feeble; still the structure does credit to the architect and builder, Mr G. Burn.

Bridge at Dunkeld.

The finest bridge in Scotland is that which has just been built by the Duke of Athol, over the river Tay, at Dunkeld. There are five large arches and two smaller land arches; the middle arch is 90 feet span, and rises 30 feet, the width over the parapets is 27 feet 6 inches. The facade has castellated turrets over the piers and abutments; the outlines of the parapets and roadway is a curve only sufficient to carry off the water; the approaches to the bridge, the Duke has rendered very complete; and the whole forms a feature, suitable to the magnificent scenery which surrounds Dunkeld. The design for this bridge was made by Mr Telford. It was executed under his directions, and finished in 1809.

From the foregoing statements, it is evident that the progress of bridge building in Britain has of late years been great; but the effects, though considerable, have been distant and unconnected. Perhaps however, the greatest and most regular scheme for opening the general intercourse through a great extent of country, that has ever been voluntarily undertaken by a free people, was that which originated in 1802. Previous to this, the northern districts of Scotland, although in sundry parts intersected by military roads, were very imperfectly opened; for these had been hastily constructed, and frequently ill fitted for the purposes of civil life.

Rapid progress of bridge building in Scotland.

In order, therefore, to encourage the spirit of improvement which had strongly manifested itself in the northern parts of the island, a board of parliamentary commissioners was established, viz.

The Right Honourable the Speaker of the House of Commons,
The Right Honourable the Chancellor of the Exchequer,
His Majesty's Advocate for Scotland,
The Right Honourable William Dundass,
Sir William Pulteney, Bart,
Isaac Hawkins Browne, Esq.
Nicholas Vansittart, Esq.
Charles Grant, Esq.
William Smith, Esq.
Charles Dundas, Esq.
John Rickman, Esq. Secretary,
James Hope, Esq. Commissioner and Law Agent in Scotland,
Thomas Telford, Engineer to the Board.

This Board, upon the application of individuals, or bodies of men, pointing out communications in the Highlands of Scotland, of public utility, and undertaking to defray a moiety of the expense, causes surveys and estimates to be made, and, if found to be truly useful, advances the other moiety from the public funds. In consequence of this, about sixty different roads have been surveyed, and a great proportion have either been already completed, or are now (1812) in a state of forwardness. Upon these roads there are bridges of different sizes, to the number of 1486, and several of no inconsiderable magnitude, viz.

Bonar bridge, of cast iron, 150 feet span, over an arm of the sea.

Dunkeld bridge, of 7 arches, viz. one of 90, two 84, two 74, two 22 feet.

Conon bridge, of 5 arches, viz. one 65, two 55, two 45 feet.

Ballater bridge, of 5 arches, viz. one of 60, two 55, two 34 feet.

Lovat bridge, of 5 arches, viz. one of 60, two 50, two 40 feet.

Wick bridge, of 3 arches, viz. one of 60, two 48 feet.

Alford bridge, of 3 arches, viz. one of 48, two 60 feet.

Potarch bridge, of 3 arches, viz. one of 70, two 45 feet.

Besides these, there are upwards of 60 bridges of one arch each, from 30 to 60 feet span.

Bridges in Ireland.

In Ireland, the city of Dublin has several fine stone bridges over the river Liffey, the principal of which are, 1. Queen's bridge, built upon the scite of Arran bridge, which had been erected in 1684, and destroyed in 1763. It was designed by Colonel Vallency, and was finished in 1768. It consists of three arches; the middle 46 feet span, the others 35 feet each; the piers are 7 feet thick, and the breadth between the parapets is 35 feet. 2. Essex bridge, which was originally founded in 1676, by Sir Humphrey Jarvis, in the viceroyalty of Arthur Earl of Essex; it being decayed, was taken down and rebuilt in 1753, from a design and under the direction of Mr George Semple, who published a very full account of the proceedings. It consists of five arches, one 58 feet span, three of 45, and one of 37 feet; the thickness of the piers on each side of the center arch is 6 feet, the breadth between the parapets 48 feet. 3. Sarah's bridge, consisting of an arch of 110 feet span, with a rise of 22 feet; the breadth between the iron railing 37 feet. This was built by Mr Stevens, an experienced bridge builder from near Edinburgh, in the year 1792. 4. Since that time Carlisle bridge has been rebuilt; it consists of three arches, the middle 50 feet span, the others 40 feet each; the thickness of the piers 10 feet, breadth between the parapets 63 feet.

There is also a very fine stone bridge over the river Lee, at Cork, built within these few years, with a draw bridge for ships at the north end of it.

Timber Bridges.

Timber bridges.

This historical sketch respecting bridges, has hitherto been confined to those constructed of stone, but

Bridge.

other materials have been successfully employed. Bridges of great extent have been constructed of wood.

With the exception of drawings made by Palladio and others, from the descriptions given in Cæsar's Commentaries, of his bridge over the Rhine, we have no satisfactory account of any ancient wooden bridge. Of those of more modern times, there is one described by Palladio, said to be situated upon the Cismone, at the foot of the Alps, between Trente and Bassane in Italy. It is of very simple construction; the whole being suspended by the framing, which forms the sides; the opening between the abutments is 109 feet. Palladio also gives sundry designs for wooden bridges formed in different ways, some of which are supported by the sides only; and one is in the form of an arch. See Plate LXXXVIII. Gautier gives designs of his own, and one from Mathourin Jousse; the last mentioned consists of one passage over it for cavalry, and another for infantry, and is protected by a roof. But the boldest and most ingeniously constructed wooden bridge on the European continent, was that at Schauffhausen, in Switzerland.

Plate LXXXVIII

Wooden bridge at Schauffhausen.

We are informed that there was formerly a stone bridge at Schauffhausen, that the Rhine injured the piers, and that in the year 1754 three arches fell; that the depth of water immediately on the upper side of the old piers being, during summer, from 18 to 20 feet, and from 28 to 30 feet below them, the idea of rebuilding a stone bridge was abandoned, and that the old piers, excepting one near the middle, were taken away; that Ulric Grubenman, a common carpenter of Tueffen, produced a model for a wooden bridge, supported only by the abutments on the banks of the river; that after some hesitation on the part of the committee of Schauffhausen, his proposal was adopted, and that he completed this truly extraordinary work in the year 1758. The total length of the bridge was 364 feet, and its breadth 18 feet. It was eight feet out of a straight line, and the angle pointed down the river; the distance from the abutment next the town to the angle was 171 feet, and from the angle to the opposite shore 193 feet. This magnificent and ingenious bridge was destroyed by the French in April 1799. See Plates LXXXIX. and XC.

Plate LXXXIX. and XC.

At Ruichenau.

About the same time that Ulric Grubenman was engaged at Schauffhausen, his brother John constructed a bridge of the same kind at Ruichenau, 240 feet in length; and some years afterwards they jointly erected one near Baden, 200 feet in length over the river Limmat.

At Baden.

We know of no wooden bridges in Europe of an extent equal to those constructed by the beforementioned ingenious men. But one upon nearly the same principles, and of 250 feet span, has been constructed over the Portsmouth river in North America, by a Mr Bludget. Yet, though of inferior magnitude, several upon principles equally simple and effective, have been erected upon rivers in Scotland by James Burn, of Haddington, in East Lothian. The largest is over the river Don, about seven miles from the city of Aberdeen, upon the road which leads from that place to Banff; the extent between the abutments is 109 feet 3 inches, and the breadth 18 feet. The frames which support the roadway are composed of short pieces of timber, but instead of being elevated above the level of the roadway in order that it may be suspended from them, they here support it after the manner of stone voussoirs. This bridge was erected in 1803. See Plate LXXXVIII.

In North America.

In Scotland.

Plate LXXXVIII

There is an elegant wooden bridge in the park at Wotton, a seat of the Marquis of Buckingham, constructed precisely upon the principle of one given by Palladio. The span of it is 87 feet, the versed sine 13, and the breadth across 20 feet.

At Wotton.

Small timber bridges, being, in all countries abounding in wood, so obvious a means for crossing streams, it is impossible to trace their origin and progress; and those consisting of rows of piles driven into the bed of a river, and supported by common trussings and bracings, being found in most countries, and being familiar to every body, it is only necessary, in what regards them, to refer to the Plates, and to what is said under the head of Practice.

Iron Bridges.

Iron bridges.

In Britain, of late years, the application of iron having been greatly extended, and practical mechanics having been also brought to much perfection, that valuable metal has been used in the construction of large arches over rivers.

At Coalbrook Dale.

The first cast iron bridge was erected upon the river Severn, about two miles below Coalbrook Dale, and between the villages of Madely and Broseley, in the county of Salop. The form of the ribs or intrados is nearly semicircular, the span being 100 feet 6 inches, and the rise from the level of the springing plates to the soffit at the middle is 45 feet, the height from the ordinary low water to the springing plate is about 10 feet, making the whole height from the low water to the soffit 55 feet. This bridge was constructed by Abraham Darby; it was cast at the Coalbrook Dale founderies, and erected in 1777. The design was bold and well executed; it formed a new æra in bridge building. The banks of the river adjacent to the bridge are exceedingly high and steep, and composed of alluvial matter which slips over the points of the coal strata. The effect of this operation not having been sufficiently provided against, some years ago, the top part of one of the stone abutments was pressed in a few inches, and of course raised up the iron work about the middle of the arch. Steps have been since taken to secure the western abutment; but the other, by having valuable houses built close up to it, is more entangled, and it may in time suffer from that cause; but the iron work has not been the least affected by the weather, or the intercourse over or under the bridge during 34 years. See Plate XCI.

Plate XCI.

At Buildwas.

The next cast iron bridge in point of time, was likewise erected upon the river Severn, about three miles above the former, at the expence of the county of Salop. Here the banks being low, Mr Telford, as county surveyor, under whose direction it was built, introduced the principle of suspending the roadway by two large ribs, one on each side of the

Bridge.

bridge. The span is 130 feet, the versed sine of the ribs which bear the covering plates is 17 feet, the breadth across the soffit is 18 feet, and the height from ordinary low water to the soffit is 34 feet. The Coalbrook Dale Company performed both the masonry and iron work by contract, and it was finished in 1796. The eastern bank of the river is composed of matter similar to that which injured the abutments of the former bridge; but here, the foundations being placed upon rock, and the masonry made of a wedge form behind, the whole has continued perfect during 15 years. See Plate XCI.

Plate XCI.

At Sunderland.

The third iron bridge, in regard to time and progressive increase of magnitude, is that erected upon the river Wear, at Sunderland, in the county of Durham. It is likewise the segment of a circle, the chord line being 236 feet, and the versed sine 34 feet: the height from the surface of ordinary low water to the soffit is 100 feet. The merit of having this bridge, instead of the ferry over the river, is chiefly due to Rowland Burdon, Esq. The iron work was cast at the founderies of Messrs Walkers, at Rotherham, in Yorkshire, and was erected under the direction of Mr Thomas Wilson. It was opened for general use in August 1796, and forms a magnificent feature and a very convenient passage in that part of the country. See Plates XCI, XCIII.

Plates XCI. XCIII.

The progress of this new species of bridge building was bold and rapid, setting out from a point, which arches constructed with stone seldom reached, that is to say, 100 feet span, extending next to 130, and from thence to 236 feet. Former experience was left far behind, and a principle introduced, to the practical operation of which we can at present assign no limit; since a design still bolder than any of the foregoing was projected, when a proposition was made by a committee of the House of Commons, for rebuilding London bridge. Mr Telford presented a plan of one arch of cast iron, the chord of which was 600 feet, and the versed sine 65 feet. This plan met with the approbation of the committee, and was by them submitted to the investigation of twenty persons, most eminent in Britain for scientific knowledge or practical skill. Their reports justified its being adopted; but a rapid succession of political events, have hitherto prevented it from being carried into effect.

Iron bridge of 600 feet span proposed over the Thames

Two very neat cast iron bridges, each consisting of one arch of 100 feet, have been built over the river Avon, at Bristol, under the direction of Mr Jessop, the engineer; and one, 150 feet span, has been designed by, and is now building under the direction of, Mr Telford, over an arm of the sea, upon one of the Highland roads, in the county of Sutherland. See Plate XCIV.

Iron bridges at Bristol.

Plate XCIV.

An iron bridge 85 feet span, has been built over the river Witham, at Boston, in Lincolnshire, designed by Mr Rennie. See Plate XCIV. (γ)

At Boston.

Plate XCIV.

Part I. THEORY OF BRIDGES.

Theory of Bridges.

The construction of a magnificent stone bridge is justly looked upon as one of the greatest performances of the masonic art: for if we compare the enormous weight of a great arch, with the strength which the cohesion of the firmest cement can give, we readily admit, that it is only by the nicest adjustment and balancing of its parts, that they are hindered from instantly falling to pieces.

Though there can be little doubt that the Romans and latter Greeks had paid some attention to this subject, from the beautiful specimens of their architecture, which exist even in our times; yet in none of their authors, either practical or scientific, is the smallest light afforded us respecting the principles upon which their practice was regulated.

The architects of the middle ages, who constructed those great cathedrals that are still the ornament of the chief cities in Europe, and the delight of the architectural antiquary, seem to have fondly indulged in the balancing of arches. They were without doubt directed by maxims, which had been elicited from a varied and extensive practice; but, whatever these were, they are to us unknown. None of these architects, though many of them were men of learning, seem ever to have committed to writing, either the history of any such erection, or the principles by which its construction was regulated. Nay, this knowledge seems rather to have been carefully kept secret, and regarded as a sort of mystery; a craft, which was only to be communicated to the *brethren*, whose experience and skill had already qualified them to be initiated into the mysteries of the sublime degree.

It does not appear, that a knowledge of this subject could be acquired otherwise than by experience. The mathematical sciences were then little known; and we may see from the construction of the bridges of that age, that the priests, who were the only architects, have had in their eye rather the successive vaulting of a Gothic cathedral, than to have originally considered of the best way of forming a permanent and convenient road. It was only about a century ago, when Newton had opened the path of true mechanical science, that the construction of arches attracted the attention of mathematicians. Since that time, volumes have been written respecting the equilibrium of arches. It has been found one of the most delicate, as it is one of the most important applications of mathematical science. Yet, with all due deference to the eminent men, who have prosecuted this subject, we are much inclined to doubt whether the greater part of their speculations have been of any value to the practical bridge builder. He is still left to be guided by a set of maxims derived from long experience, and as yet little improved by theory. In truth, his works seldom fail even where they differ farthest from the deductions of the theorist; and at all events, he finds that a much greater latitude is allowable than theory seems to warrant. He is therefore surely excusable

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in doubting of the justice of such theories, at least until they are more consonant to the approved practice.

A new theory of arches proposed.

It is our intention, in the present article, to point out a new mode of considering this subject, to which, with great diffidence, we request the attention of the intelligent practitioner. It may indeed still be deficient, if not in some respects erroneous. But it will, we think, have this merit, that of being readily apprehended, and easily applied, without requiring much previous scientific information. Indeed though we highly value the sublime geometry, we are inclined to think that the unnecessary parade of calculus in the application of science to the arts, has been one of the chief causes of the dislike, which many able practical men of our country have shewn to analytical investigation.

Common theory of equilibration.

Nevertheless, as many of our readers are well qualified to comprehend, and will naturally expect that we should point out, the modes of investigation, usually pursued in this interesting subject; we shall previously, and in as succinct a manner as possible, endeavour to lay before them the commonly received theory of equilibration. From which, having cleared away the useless rubbish, if we can extract any proper materials, we may, like œconomical builders, make good use of them in our future structure.

Catenarian curve proposed by Dr Hooke.

The first thing like a principle that we meet with is in the assertion of the eminent Dr Hooke, that the figure into which a heavy chain or rope arranges itself, when suspended at the two extremities, being the curve commonly called the *catenaria*, is, when inverted, the proper form for an arch; the stones of which are all of equal size and weight.

Plate LXXX. Fig. 2.

Now, as this idea, strictly just, has been very generally adopted, and affords some useful hints, it may be well worth while to examine it. Let A, B, Fig. 2. be a string or festoon of heavy bodies, hanging by the points A, B, and so connected, that they cannot separate although flexible. These bodies having arranged themselves in the *catenaria* ACB, conceive this to be turned exactly upside down. The bodies A and B being firmly fixed, then each body in the arch ADB, being acted on by gravity, and the push of its two neighbours with forces exactly equal and opposite to the former, must still retain its relative position, and the whole will form an arch of equilibration.

This arch, however, would support only itself; nay, a mere breath will derange it, and the whole will fall down. But if we suppose each spherule to be altered into a cubical form, occupying all the space between the dotted lines, the stability will be more considerable. And as the thrust from each spherule to its neighbour is in a direction parallel to the tangent of the arch at the point of junction, it is obvious, that the joints of our cubical pieces must be perpendicular to that, so as to prevent any possibility of sliding.

Our arch is now composed of a series of truncated wedges, arranged in the curve of the *catenaria*, which passes through their centres; and we are disposed, with David Gregory, to infer, that when other arches are supported, it is only because in their thickness some *catenaria* is included.

We might pursue this subject a great deal farther, by investigating all the useful properties of the *catenarian* curve: but, in our opinion, this is at present unnecessary. This curve is, indeed, the only one proper for an arch consisting of stones of an equal weight, and touching in single points, but is not at all adapted to the arch of a bridge, which, independent of the varying loads that pass over it, must be filled up at the haunches, so as to form a convenient road-way. In this case, some farther modification becomes necessary. The *haunch* E of the arch ACB, bearing a much greater depth of stuff than the crown, it must be so contrived as to resist this additional pressure. Every variation of the line FGH, or *extrados*, will require a new modification of the curve ACB, or *intrados*, and the contrary. Accordingly, M. de la Hire has suggested a good popular mode of investigating this subject. Let it be required to determine the form of an arch of the span AB, and height CD, proper for carrying a road-way of the form FGH. Mark off, upon a vertical wall, the points A, B, C′, inverting the required figure: Suspend from A, B, a uniform chain or rope, so that its middle may hang a little below the point C′, and dividing the span A B into any number of equal parts, and drawing the perpendiculars *a b*, *c d*, &c. suspend from the intersections *e*, *f* bits of chain *e b*, *f d*, &c. so trimmed, that their ends may fall on the line of road-way; and it may be observed, that as those pieces, which hang near the haunch, will bring it down, the crown C will thereby be raised into its proper position.

Plate LXXX. Fig. 3.

Popular method of De la Hire.

All will now do, provided that the sum of the small pieces of chain has to the large one, A C′B, the same ratio which the stuff to be filled into the haunches has to the whole weight of the archstones; the depth of which must of course be previously determined. But, if this is not the case, it will be easy to calculate how much must be added to, or subtracted from, the small chains, in order to obtain this proportion. This being equally divided among the small chains, will give a road-way very nearly parallel to the former. The curve will evidently be a perfect curve of equilibration, and extremely near the one wanted. And this whole process is so easy, that it may be gone through in a short time by any intelligent mason.

Mathematical theory of equilibration.

But although this mechanical way of forming an equilibrated arch be founded upon principles sufficiently just, and be perhaps the simplest and best way in which the practical builder could form the original design of such an arch, yet as it affords no general rules that may be applied to the construction of arches, we proceed to consider the same subject in a mathematical point of view.

Fig. 4.

And first, then, in the semicircular polygon, as it is called, Fig. 4, where weights are hung on the thread A C′C C″B, which bring it into the position ACB, we have at each angle three forces in equilibrio. Wherefore, by the principles of statics, they are to one another as the sines of the opposite angles; that is, the tension *r* C is to the tension *l* C, as sine *l* CW is to sine *r* CW, but the tension from C to *l* is the same as from C′ to *r*. Also sine *l* CW is the same as sine *r′* C′W′, since these angles are supplementary, CW, C′W′ being parallel; therefore the tension *r* C is to the tension *r′* C′, as sine *r′* C′W′ to sine *r* CW. *Or, the tension*

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Again, we have as sin. d C l : sin r C l :: tension rC : tension d c$=\frac{r\text{C}\times\sin.r\text{C}l}{\sin.d\text{C}l}$; but as r C is inversely as sine r Cd, therefore tension d C is as $\frac{\sin.r\text{C}l}{\sin.r\text{c}d\times\sin.d\text{C}l}$.

Plate LXXX. Fig 5. Now, let there be an unlimited number of weights hung from the chord, and indefinitely near each other, our polygonal thread becomes a curve, Fig. 5. being in fact the curve of equilibration adapted to the weight which depends from it. The angles rCd and lCd become r'Cd and l'Cd, which are supplementary, and have equal sines, wherefore the product of these sines is the square of either. Also, as the sine of rCl or rCr' is as the curvature, or reciprocally as the radius of curvature, we have tension dC, or weight on C, inversely as rad. curv. × sin.^{2} inclination to vertical.

This tension, in the present case, is usually produced by the gravity of the superincumbent materials, and may be measured by the area contained between two indefinitely near vertical lines, EF, ef, Fig. 5; but while the distance Ee is constant, the area Fe will diminish with the sine of EFe, as Ee becomes more upright. To countervail this, we must enlarge the depth EF in the same proportion as sine eEF diminishes. And, therefore, we have EF inversely as rad. curv. × sin.^{3} eFE. *That is, the height of the superincumbent matter must be inversely as the radius of curvature, into the cube of the sine of the inclination of the curve to the vertical.*

This, then, is the leading principle of the commonly received theory of equilibration. The mode in which we have derived it is concise, but we trust it will not be found the less clear, or the less easily apprehended.

Let us proceed to apply the theory to some practical cases.

If the arch be the segment of a circle, then the radius of curvature is the same throughout, and the height will be inversely as the cube of the sine of inclination to the vertical. And from this we derive the following very simple construction, for describing the equilibrating extrados of a circular arch, and which the reader, who has examined this subject, will find much easier than those commonly given.

Fig. 6. At any point D, draw the vertical Dd, and DF from the centre C; then laying off D a equal to the thickness at the crown, draw the perpendiculars ab, bc, cd successively, Dd is the vertical thickness at D, or d is a point in the extrados.

For it is evident, that Da : Db :: Db : Dc : Dc : Dd, because of similar triangles; therefore Da : Dd :: rad. sec.^{3} aDb, or inversely as radius to cube sine $a$$b$D. Now D$a$ is the thickness at crown, and Db is therefore the thickness at D. Figure 7 is constructed in this way, and may serve as a specimen of the equilibrating extrados for a semicircular arch. By reversing this operation, we may find the thickness at the crown corresponding to a given thickness at any other point. And here we may observe, that as D approaches the extremity B of the semicircle, the line Dd rapidly increases, until at the point B it is of an infinite length. But indeed this must evidently be the case with every arch which springs at right angles with the horizontal line; for the thrust of the arch should be resisted by a lateral pressure, and no vertical pressure can act laterally on a vertical line.

Fig. 7.

Fig. 6.

We may also observe, that since the extrados or upper outline descends first on each side of the crown, and then ascends with an infinite arc, there is, for any thickness of the crown, a point on each side where the upper edge of the extrados is on a level with that on the crown. Thus, if BD$=30°$, its sine is half the radius. Da is therefore $=\frac{1}{8}$ of Dd, so that if Vv=Da be made $\frac{1}{11}$ of VC the radius, we have the point d at the same level with V. Between this point, however, and the crown, there is a considerable depression, which is increased if the crown be made still thinner. On the other hand, if it be made thicker, the horizontal line drawn through the crown cuts the extrados much nearer the middle of the arch. It appears, therefore, that the circle is not well adapted for the purposes of a bridge, or a road, where the roadway must necessarily be nearly level; for no part of the extrados of the circular arch will coincide with the horizontal line. There is indeed a certain span, with a corresponding thickness at the crown, where the outline differs least from the horizontal; that is, an arch of about 54 degrees, with a thickness at the crown about $\frac{1}{4}$ of the span. But that is far too great for practical purposes.

Fig. 6.

We may, however, extend the construction just given, even to those arches that are formed of portions of circles differing in curvature. For the equilibrating extrados being first constructed for that portion of the arch in which the crown is, as far as the vertical line passing through the contact of the neighbouring curves, the thickness of the crown must be supposed to be enlarged, in proportion to the diminution of the radius of curvature, or the contrary, and, with this, proceed as before along the succeeding branch of the curve. This will, indeed, cause an unsightly break in the extrados, for which we shall not at present pretend to find any other remedy, than using materials of a different specific gravity.

Those who wish to examine this subject farther, may consult Emerson's *Fluxions*, or Hutton's *Principles of Bridges*. We shall only observe here, that the extrados of the ellipse, and of the cycloid, resemble that of the circle, having an infinite arc on each side at the springing; and indeed this, as has already been observed, is a general rule for all those curves which spring at right angles to the horizon. In the parabola, the extrados is another parabola exactly the same, only removed a little above the other. In the hyperbola, the extrados is another curve, which approaches the interior arch towards the springing. None of these curves, therefore, can, with propriety, be employed for the arches of a bridge, though there may be cases where a single arch might with propriety be formed into a conic section.

The catenaria, which has been much spoken of as the best form for an arch, has an extrados, the depression of which, below its crown, at any point, is to the depression of the curve in the same vertical line, in a constant ratio. This ratio is that of the constant tension at the vertex, to the same tension diminished by the thickness or vertical pressure in

Theory. the crown. If the vertical pressure be less than the tension, the extrados falls below the horizontal line; if greater, it will rise above it.

Mathematicians finding the circle and other common curves so little adapted to the arch of a bridge, which has a horizontal roadway, have, in the next place, endeavoured to solve the converse of the problem, and give a rule for finding the intrados or figure of the arch, which have the exterior curve a horizontal line.

This problem can only be resolved by calling the fluxionary calculus to our aid. It is a case of the more general one to find the intrados, when the extrados is given; and being the most useful case of that problem, fortunately admits of a solution comparatively easy.

Method of finding the intrados when the extrados is given.

We have already seen, that the load DC is inversely as rad. curv. × sine³ inclination to vertical. Calling therefore, as usual, the abcissa VE$=x$, CE$=y$, VC$=z$, we have $cf=\dot{x}$, $fe=\dot{y}$, $Cc=\dot{z}$; and since $Cc : cf$ :: rad. : sin. inclin. at C; therefore the load DC is inversely as rad. curv. $\times \frac{\dot{y}^3}{\dot{z}^3}$. But, as is well known, the rad. of curvature$=\frac{\dot{z}^3}{\dot{y}\ddot{x}-\dot{x}\ddot{y}}$; therefore, by multiplication, DC is inversely as $\frac{\dot{y}^3}{\dot{y}\ddot{x}-\dot{x}\ddot{y}}$, that is, directly as $\frac{\dot{y}\ddot{x}-\dot{x}\ddot{y}}{\dot{y}^3}$, or as $d\left(\frac{\dot{y}}{\dot{x}} \div \dot{y},\right)$ and is equal to $d\left(\frac{\dot{y}}{\dot{x}} \times \frac{C}{\dot{y}},\right)$ where C is a constant quantity, found by taking the real value of DC at the vertex V of the curve.

Plate LXXX. Fig. 8.

Now, in the present case, calling AV$=a$, we have DC$=a+x$, (AV+VE,) therefore $a+x=\frac{C}{\dot{y}} \times$ flux. of $\frac{\dot{y}}{\dot{x}}$. Take $\frac{\dot{x}}{\dot{y}}=u$, and by integrating, we have

$$u=\sqrt{\frac{a2x+x^2}{C}}, \text{ and therefore}$$

$$\dot{y}=\left(\frac{\dot{x}}{u}=\right)\sqrt{C}\times\frac{\dot{x}}{\sqrt{2ax\times x^2}}; \text{ whence by integration}$$

$$y=\sqrt{c}\times \text{Lo.}\left(2ax+2x^2+2\sqrt{2ax+x^2}\right)+B.$$

At the vertex $x=0$, therefore

$$y=\sqrt{C}\times \text{Lo.}(2a)$$

And consequently the ordinate

$$y=\sqrt{C}\times \text{Lo.}\frac{a+x+\sqrt{2ax+x^2}}{a}.$$

Lastly, to find the value of $\sqrt{C}$, we take some point of the extrados, where the ratio of x and y is known. For example, if the span $=2S$, and height $=h$ are given, we have

$$S=\sqrt{C}\times \text{Lo.}\frac{a+h+\sqrt{2ah+hh}}{a},$$

$$\text{hence } C\sqrt{}=S\div \text{Lo.}\frac{a+h+\sqrt{2ah+hh}}{a},$$

$$\text{and finally } y=S\times\frac{\text{Lo.}\,\frac{1}{a}\left(a+x+\sqrt{2ax+x^2}\right.}{\text{Lo.}\,\frac{1}{a}\left(a+h+\sqrt{2ah+h^2}\right.}.$$

We subjoin a Table, calculated by Dr Hutton from this formula, for an arch of 100 feet span and 40 feet rise, the thickness of the crown being taken at 6 feet. It is nearly of the same dimensions as the middle arch of Blackfriar's Bridge, and which may answer for any arch where these dimensions are similarly related to each other.

EC.	DC or AE.	EC.	DC.	EC.	DC.
0	6.000	21	10.381	36	21.774
2	6.035	22	10.858	37	22.948
4	6.144	23	11.368	38	24.190
6	6.324	24	11.911	39	25.505
8	6.580	25	12.489	40	26.894
10	6.914	26	13.106	41	28.364
12	7.330	27	13.761	42	29.919
13	7.571	28	14.457	43	31.563
14	7.834	29	15.196	44	33.299
15	8.120	30	15.980	45	35.135
16	8.430	31	16.811	46	37.075
17	8.766	32	17.693	47	39.126
18	9.168	33	18.627	48	41.293
19	9.517	34	19.617	49	43.581
20	9.934	35	20.665	50	46.000

The curve of Fig. 8. is accurately drawn to these dimensions, and may give an idea of the form of an equilibrated arch. It is not destitute of grace, and is abundantly roomy for craft. Plate LXXX. Fig. 8.

Such, then, is the analytical theory of equilibration: for a practical subject it does, we confess, appear abstruse.

Those who have already studied the theory, will observe, that we have greatly simplified the investigation. The construction we have given for circular arches we shall probably find useful hereafter. We could with pleasure have prosecuted the subject farther, not only as it affords some good general views of the equilibration of arches, but exhibits also several beautiful examples of the application of the higher calculus. Yet we must repeat, with all due respect to the learned and eminent men who have turned their attention to it, that we fear their speculations have been of little value. In saying this, we do not mean to surmise, that their deductions are any way erroneous; they are legitimate consequences from the principles assumed. But it appears to us, that the writers on equilibration, like many others who have hastily applied analysis to physics, have taken too narrow a view of their subject to comprehend all the variety of practice. Setting out with one leading principle, best adapted, perhaps, to the application of calculus, they neglect the numerous circumstances by which it may be modified, and which are too important to be overlooked in drawing practical inferences from such an investigation.

Their principal care respects the figure of the soffit, a thing which the practical engineer knows may admit of the greatest variety. As to the thickness of archstones, side wall, and piers, the horizontal section or ground plan of the bridge, the manner of filling up its haunches, of forming the joints, of connecting it with the abutments, wing walls, &c. we are still left in the dark.

The analytical writers have assumed one leading

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principle, that the arch is in every point kept in equilibration solely by the gravity of the superincumbent column of matter. Now, it is even doubtful whether this principle be true. At any rate, they do not consider the numerous modifications which it receives, from the cohesion of that matter among itself; from the mutual cohesion and friction of the archstones; from the position of their joints; from the different specific gravity which the arch and superincumbent matter have, or which they may be made to have; from the lateral, and in some cases hydrostatic pressure, propagated to the masonry throughout that matter; and, in fine, from a number of other causes, which, if not singly, are, when combined, at least of as much importance as the gravity of the vertical column of matter alone.

Method of De la Hire, Parent, Belidor, and Atwood.

Let us turn, therefore, to another mode of considering this subject, which has been adopted by De la Hire, Parent, Belidor, and many others on the continent, and in our own country by the ingenious Mr Atwood.

The latter has, from the known properties of the wedge, and the elementary laws of mechanics, exhibited to us a geometrical construction for adjusting the equilibration of arches of every form. The mathematical reader, who has not lost his relish for the ancient geometry, will find there an elegant specimen of its application; for he completes his geometrical construction without once having recourse to any other than the principles of elementary geometry and trigonometry. It had been well, indeed, if he had adhered longer to that mode of investigation; for, by applying the analytic form too early, he has been led unawares to consider that only as an approximation to the values of the quantities sought after, which, in fact, is the expression for the values of these quantities themselves. Nevertheless we owe much to Atwood: he has shewn, that the advantages of equilibration are not confined to any particular curve; that the drift or horizontal thrust of an arch may be easily found; and that an arch may have all the advantages of equilibration, whatever its figure may be, merely by adjusting the joints of the archstones.

The stones, or sections of an arch, being of a wedge-like form, have their tendency to descend opposed by the pressure which their sides sustain from the similar tendency of the adjoining sections. Should this pressure be too small, the stone will descend; should the pressure be too great, the stone will be forced upwards.

These pressures act in directions perpendicular to the touching surfaces; for, if the original direction of any pressure should be oblique, it may be resolved into two forces, of which, while one is perpendicular to the surface, the other is parallel to it, and, of course, neither increases nor diminishes the perpendicular pressure.

Plate LXXX, Fig. 9.

The wedge A, Fig. 9, if unimpeded, would descend in the direction VQ, but is prevented by the reaction of B and B′, acting in the directions PQ and K′I, perpendicular to the sides AG, QD; and it is known, from the properties of the wedge, that if PQ, or K′I be to the weight of the wedge A, as DO is to DG, the wedge A will remain at rest. If also the wedge A be only at liberty to slide down GA, considered as a fixed abutment, then the force PQ alone will keep it in equilibrio. The force PQ being perpendicular to DO, has no tendency to make A slide either up or down on that line, but produce it towards N, making NM equal to PQ; then this force acting obliquely at N, may be reduced to two others, viz. MR perpendicular to AG, expressing the perpendicular pressure on the abutment of A, and RN expressing the force or tendency it has to make A slide upwards along AG. Again, take the vertical line A *a*, expressing the weight of A, and draw *a* H at right angles to AG; it is very evident, that AH expresses the tendency of A by its weight to slide down GA. AH is opposite, and is equal to NR.

For, draw the perpendiculars D *d* and A *p*, then the triangles A *a* H, AG *p*, DG *d* are evidently similar; and also the triangles OD *d*, OQN, MNR, as they have always a common angle besides the right angle. Now the force PQ, that is, MN is to the weight of A, that is A*a*, as OD to DG by supposition.

And A *a* : AH :: AG : A *p* :: DG : D *d*
Therefore, MN : AH :: OD : D *d* :: MN : NR.

Or MN has the same ratio to AH, that it has to NR; that is, AH and NR are equal, or the tendency of A to slide downwards by its weight, is balanced by the tendency of MN to make it slide upwards: wherefore the section A remains at rest in equilibrio.

Considering the whole arch as completed, with its parts mutually balancing each other, the force PQ, which is necessary for sustaining the wedge A, will be supplied by the reaction of the adjacent wedge B. Now, let it be required to ascertain the weight of B in proportion to A, so that they, being adjusted to equipoise, may continue to be in equilibrio, when left free to slide along KB. Since MR is the pressure produced by PQ in a direction perpendicular to AG, we must add to this H *a*, which is derived from the wedge A; therefore make M *n* equal to H *a*, produce MR to Y, take YZ equal to R *n*, draw ZW at right angles to KB; YW is the force tending to make B slide up BK: take therefore BH′ equal to YW, draw the perpendicular H′ *b* meeting the vertical B *b* in *b*; B *b* will represent the necessary weight of the wedge B; and the whole is so evident from the composition of pressures, as to require no further demonstration. Such is Atwood's construction; he has rendered the demonstration much more prolix, by the unnecessary introduction of trigonometry; and after shewing how the weight of the sections C, D, &c. may be found in the same way, he goes on to reduce these weights and pressures to analytical and numerical values. He finds these in terms of the sines and tangents of the successive angles of inclination; but in reducing these to numbers, he has been led to the accumulation of small errors in that very operose way of proceeding, to give erroneous results; and into the singular mistake of conceiving, that the real expression of these values was only an approximation. Had he recalculated the whole by more extended trigonometrical tables, they would have quickly undeceived him; and they would have shewn him, that what he was thus searching so deeply for, was all the while lying exposed at the surface; that the apparent difficulties were entirely of his own creation, and his imagined accuracy was error. This should teach mathemati-

Theory. cians to beware of thinking calculation the surest mode of eliciting truth. It should be the last thing employed. Nothing is so simple, so perspicuous, as the diagram. Its geometrical properties should be pursued as far as possible. They are not only clear, they are palpable. And in such applications of mathematical theory, the whole being a creature of the mind, it seldom admits of an approximating value in any part.

Let us now return to the geometrical construction.

The weight of the section C, may be determined in the same way as the foregoing. But surely more simply thus: From C draw CS parallel to WZ, that is at right angles to KO, and make it equal to WZ + H′ *b*; draw S*c* at right angles to LO, meeting the vertical C*c* in *c*, then C*c* represents the weight of C. From D, draw DT parallel and equal to S*c*, draw T*d* perpendicular to DO, meeting the vertical D*d* in *d*, D*d* is the weight of D, and so on successively.

Nay, instead of drawing DT parallel to S*c*, and T*d* perpendicular to DO, we may at once draw from S, S*d*′ perpendicular to DO, which will cut off for us *c d*′ = D*d*, the weight of the section D. It is of no consequence, although the lines of abutment do not all run to the same centre O.

Fig. 10. And thus we obtain a general construction for all the sections, which turns out abundantly simple, Fig. 10: for, upon any vertical line *b*′ *e*, if *c b* be taken to represent the given weight of any section C, and *c* T be drawn at right angles to CO, and *b* T at right angles to BO, meeting the other in T: then T*b* represents the pressure against the abutment OB, and TC the pressure against O*c*, and by drawing T*d* at right angles to DO, T*e* to EO, &c. we have the weights of the successive sections represented by *c d*, *d e*, &c. and the pressure on their lower abutments represented by T*d*, T*e*, &c.

We may carry the same mode of determination to the other side of C, and pass the vertex of the arch. The divisions representing the weights of the sections will run upwards along the indefinite line *c b*′. The pressures on the abutments will be determined as before. Should the two sides of a section be parallel, the perpendiculars through T upon them will coincide; such a section therefore should have no weight. But should the two lines of abutment diverge towards the lower side, the line expressing the weight of that section will return upon the vertical, shewing that such a section requires the reverse of weight, viz. a support from below. The line TV drawn horizontally through T exhibits the horizontal pressure, which is uniform through the same equilibrated arch. But it is evidently greater, the less *b* T and *c* T are inclined to each other, the weight *b c* being constant, that is, the smaller the angle of the wedges or sections. It also increases directly as the weight of the section C, &c. The line V*e* expresses the weight of the semi-arch or perpendicular pressure on each pier; being the sum of the weights of all the sections in the semi-arch.

Again, it is obvious that the angles *b* T *c*, or *c* T *d*, &c. are equal to the angles of the sections BOC, COD, &c. If therefore the weight of any section E be given = *de*, and the requisite angle of that section be required, every thing else being known, we have only to join T*e*, and the line EO being drawn perpendicular to T*e*, will exhibit the inclination of the lower abutment of the section; *d* T *e* is the angle of that section. And here it matters not where the point E be, that is, how great the base of the section be, provided the weight is equal to *de*. We also see that while the angles remain the same, and the weights proportional, it is of no consequence what the curve passing through the lower edges of the sections, or through their upper edges may be, they may even be straight lines. According to this principle, *the architect is not confined to given forms of intrados or extrados*; he may take whatever curve appears most beautiful or useful: and, what is more, by the proper adjustment of the joints, *he may cast the ultimate pressure in any direction which he thinks most conducive to the strength of the edifice.*

The reader will easily perceive, that the segments of the vertical line rapidly increase, as the perpendiculars to the line of abutment approach the vertical; that is, as the abutments approach the horizontal line; and in that position, the last segment becoming infinite, it is impossible by mere weight alone to effect the adjustment of the sections.

Though the geometrical construction we have just given is so simple, that it appears likely to answer every practical purpose; yet it may be proper to express analytically, or rather arithmetically, the values of the several quantities concerned in the investigation. This is attended with no difficulty, as T V *e* being a right angled triangle, it is obvious that the weight V*e* of the same arch is the tangent of VT*e*, or of the inclination of the lower abutment, when TV the horizontal force is radius; at the same time also, the pressure T*e* on the abutment is the secant of the same angle; and the weight *cd* of any section is the difference of the tangents of the inclinations of its upper and lower abutments. In like manner *a* V, the weight of half the key-stone, is to TV the horizontal force as the tangent of half the angle of that section is to the radius; or, as radius is to the cotangent of the same angle.

Application to the flat arch. Plate LXXXI. Fig. 1. We now proceed to shew the application of this investigation to some practical cases, and the first we shall consider, is that known by the common, though aukward name of the *flat arch*; one with which every mason is perfectly familiar, though it be seldom noticed by writers on equilibration. AB*b a* is a structure of this kind, adjusted to this equilibrium, and resting on the abutments A*a*, B*b*. Its construction is exceedingly simple; nothing more is necessary than to draw all the joints *m*M, *l*L, &c. to one centre C; and the reason is obvious; for DK, KL, &c. are the differences of the natural tangents of the inclinations of the abutments, the perpendicular CD being radius; and the same thing is true in the line *da*, and in every other parallel section. The surface therefore A*m*, M*l*, that is, the bulks or weights of the stones, are in the same ratio, and it is that which is required by the above principles. Also, if we assume the line of its base to represent the weight of any stone in the arch, for example, KD for half the keystone; then the perpendicular CD is the horizontal *thrust*, *drift*, or *shoot* of the arch. By increasing DC, or diminishing it, that is, by drawing the joints to a lower, or a

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higher centre, we may alter this thrust at pleasure. What if we should take C up to D? Some curious ideas occur here, but being chiefly speculative, we shall not now pursue them. They serve to connect this case very neatly with the lintel and the Egyptian arch, (or that formed by flat courses of stones gradually overlapping each other, until the opening be covered), in each of which the horizontal thrust vanishes. We ought also to observe, that whatever weight of stuff lies on an arch of this kind, there is no change of design requisite, so long as the upper surface or roadway is horizontal. For being every where of the same height, the mass incumbent on any stone will be proportional to its base, viz. the back of that stone; since we must conceive the stuff to press vertically. It is therefore the same as if the whole arch had undergone a change of specific gravity; every pressure will be increased in the same proportion.

Plate LXXXI. Fig. 1.

The design of an equilibrated horizontal arch, or plat band, being thus easily formed, it will not be difficult to extend it to a curve of any form, *a ∂ b b′ d′ a′*, Fig. 1. is an arch of this kind. It is a circular segment from the centre C, to which the joints of the horizontal arch were directed; the two key stones have the same weight and obliquity of abutment; consequently the horizontal thrusts are the same. The other arch stones being previously intended to have the same weight with those of the flat arch, it is only necessary to draw the lines 1 1, 2 2, 3 3, parallel to K*k*, L*l*, M*m*, and so as to produce this equality. This being merely a simple problem in mensuration, we shall not occupy the reader's attention with the solution of it. In the Figure referred to, we have divided the soffit AB of the flat arch into equal parts; all the stones therefore of that as well as the curvilineal form, are of equal magnitude and weight, the angles of the arch stones only varying. We might make a table of these angles, to any given form of keystone, but it is really unnecessary; for we have only to take the tangent of half the angle of the key stone, or more correctly, of the angle of inclination to the vertical of one abutment of the keystone, from a table of natural tangents, and by adding to it twice the same number successively, we have the natural tangents of the inclinations of all the other abutments. We believe, however, that the practical builder will prefer a geometrical construction to this, and lay off his joints by means of the common bevel.

Before we take leave of the straight or flat arch, there is another of its properties we would wish particularly to be noticed. The reader must have already observed, that when CD expresses the horizontal thrust, or pressure of the vertex, CK, CL, CM, &c. express the perpendicular pressures on the successive joints K*k*, L*l*, M*m*, &c. Now, it is obvious, that K*k*, L*l*, &c. are proportional to CK, CL; for AD, *ad*, are parallel. Therefore the vertical sides of the arch being parallel, *the pressure on each joint of the flat arch is always proportional to the surface of that joint*, and the pressure on each square inch of joint throughout the arch is always the same. It may readily be found too, by dividing the horizontal thrust by the area of the vertical section D*d*. This is a most valuable property, for it secures uniformity of action in every part of the structure. But it is not to be found in the arch *abd*; for there, the joints being nearly equal, the pressure on each increases as we descend from the vertex, and may, at the lower sections, be eventually so great as to overcome the cohesion of the materials.

It may be objected to the straight arch, that the acute angles, as A*am*, AM*m*, are very apt to chip away, and weaken the arch. Now this is certainly true, but it has no connection with the doctrine of equilibration. There is, however, a very ingenious mode of remedying it; for if the upper and lower extremities of each joint be drawn to a centre, considerably below the former, or even be formed into vertical lines, as at *m*, *n*, it will materially stengthen the acute corners without injuring the equilibration. We may conclude, therefore, that a structure of this kind possesses every requisite that can be looked for in an equilibrated arch. Is the flat arch, then, which admits, with such facility of the most perfect equilibration, one of the strongest possible figures? We believe every practical man can give us a prompt answer to this question. But, before we take any farther notice of it, we shall proceed somewhat farther with the applications of our theory. The segment *a ∂ b* was adjusted to equilibrium, with reference to the flat arch, upon the principle that the weight of the archstones was only to be provided for. In general an arch of this kind is filled up at the flanks, so as to form a roadway as nearly as possible horizontal. We must, in that case, when considering the weight of each archstone, not lose sight of the difference of pressure upon it, arising from the varying height of the incumbent mass. Having, therefore, divided the back of the arch into sections *d* 1, 1 2, 2 3, Fig. 2, each containing one, two, or more arch stones, and having drawn the vertical lines from these divisions to the line of roadway, we calculate the weight of the trapezoid of the stuff over each section; add this to the weight of the section; and divide the tangent line or flat arch accordingly.

Plate LXXXI. Fig. 2.

We may even give a construction for this. The stuff over any section 2 3, is proportional to the trapezoid *t* 2 3 *v*, or nearly *tv* × *sw*; for we need take no notice of the small segment of the circle between 2 and 3, but consider the arch as polygonal, in which case the mean height is *sw*.

But 1 2; 2 3 being equal, we have *tv* or 2*y* as sine of 2 3 *y* (*i. e.*) as sine of the inclination of the arch; wherefore, drawing the mean height *ws*, and producing C*w* to meet the perpendicular *sx*, take the weights over the sections to be represented on the horizontal line, by lines equal to *wx* respectively; for *sw* is to *wx* nearly as 2 3 is to 2*y*, and *tv*, at the vertex of the arch, is equal to 2 3; and since the weight of the archstone will be nearly constant, and that on the supposition that the weight over each section is represented by the trapezoidal space included between it and the roadway, let us assume the weight of the keystone, as represented by the part *d*P, and the others by similar additions. If we have an arch differing in gravity from the stuff which loads it, we can measure to a circle within, or without the circle of intrados PT*u*W. Draw, therefore, the horizontal line P*o*, and lay off P*a* equal to $\frac{1}{2}$ P*q* for the half key-

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This method is evidently only an approximation; we consider the principal load as arising from the mass incumbent on each section, or at least that the weights of the sections are proportional to these masses. It becomes pretty accurate, by taking w in the mean circle drawn between the soffit and back of the arch; and we might render it still more accurate, by giving the determination a fluxionary form, but we write at present for the practical builder, to whom the calculus is seldom known; besides, as the reader will see hereafter, we do not think the rigid determination of this matter as yet of much consequence.

Having thus discovered the weights of the sections, and laid them off on the horizontal line, as if for a flat arch, and having, either from the given form of the keystone, or the horizontal thrust, drawn the angles of abutment which a flat arch would require, the joints of the arch in question are to be drawn parallel to these, and through the extremities of the proper sections, previously marked out, as above mentioned. If there be intermediate joints, they may either be drawn properly related to the others, or separately, discovered by a repetition of the construction. For example, let C be the given centre for the keystone; draw Ca, Cb, Cc, &c.; and through 1 draw the joint 1R parallel to Ca, also 2T parallel to Cb, and 3W to Cc, &c.: the arch would then be in equilibration. Plate LXXX. Fig. 2.

Thus we find, that, *by the proper adjustment of the joints to the weight of the section, we may form equilibrated arches, having soffits of any figure* that may be thought proper, and with any proportion of dead weight over them that circumstances may require. Let us now look at the converse of this problem; where the inclinations of the joints being given, it is required to discover the mass or weight which must be allotted to each section, so as to preserve the whole in equilibrium.

Pursuing the mode already employed, it is evident, that if we lay off from one centre the angles to be formed by the successive joints, or abutments, with the vertical line, a horizontal line drawn to cut them will represent, by its successive segments, the weights of the several sections; while, at the same time, the perpendicular let fall from the centre on this line will exhibit the horizontal thrust. If the arch, therefore, must throughout be of equal thickness, we have only to mark off upon the soffit, or rather upon the mean curve, segments proportional to those of the horizontal line. If the upper and lower outline of the arch be determined, we must divide it into trapezoids, having the same proportions; then draw the joints parallel to the lines expressing the given angles of inclination. Such joints will run to several different centres, thereby shewing us, that their union in one is not at all necessary to the security of the arch, even should that be a portion of a circle.

The position of the joints is usually given in a different way from that which we have just considered. In circular arches they are generally formed by producing the radii from the centre; and in others they are commonly drawn perpendicular to the curve. Now, though we have just shewn, that this is by no means necessary to the equilibrium, yet, as it is in reality the most convenient in practice, it may be of importance to attend to the effects likely to be produced by this modification. Theory.

We see, in Fig. 10. that the tangents on the horizontal line rapidly increase as we pass outward, and we should therefore increase, in the same proportion, the weight of our sections. We cannot increase the base as proposed above, for that is necessarily given by the position of the joints, but, as we are still able either to increase the height or the breadth of the sections, we may consider the effect of both these modes. Plate LXXX. Fig. 10.

Let it be required, then, to equilibrate a circular arch, where the stones being all of equal thickness, with joints equally distant, and drawn all to one centre, we are only at liberty to increase the width of the roadway, or length of the horizontal courses.

Considering each course of arch stones as a prism of a given base, a supposition sufficiently accurate, it is evident, that its magnitude or weight increases with the length only. But this weight must, from the principles already laid down, be as the difference of the tangents of its abutments; the length therefore must be in that ratio. Accordingly we find the breadth at different distances from the vertex in the same way with the weights of the sections: the breadth at 45° must be double, and at 55° must be about triple of that at the crown, and will increase still more rapidly afterwards. Proportions such as these may answer well in the short flight of steps for a flying staircase, but are quite unfit for our present purpose. When we recollect, however, that in a bridge, the extraordinary expansion towards the haunches is materially corrected by the increased pressure of the incumbent mass in that part, we are encouraged to proceed a little farther, and consider the effect of the second mode of effecting the equilibrium.

The pressure of matter upon each section has already been stated as proportional to $tv \times sw$; but tv is the difference of the sines of the angular distances of the successive abutments from the vertex, and sw is the mean versed sine added to the given thickness at the crown, when the roadway is horizontal. We have therefore the pressure as the difference of the sines $\times$ (mean versed sine $+$ thickness at vertex.) But these pressures are also, from the theory, as the difference of the tangents of these angular distances. In the present case, where the angles of abutment, and, consequently, where the difference of their sines and tangents are known, and where the mean versed sine may also be readily formed, it will not be difficult to state the conditions of equilibrium for an arch of any dimensions. Plate LXXXI. Fig. 2.

In the common mode of building, we must give the arch a sufficient thickness at the keystone, to resist the horizontal thrust, ensure stability, and bear the loads likely to come upon it. We must also cover this part with a certain thickness of gravel, or other matter, so as to form a roadway. The varying pressure of the wheels of a loaded carriage, when it is propogated through this stratum of gravel, will be so far diffused as not to disturb the stone immediately be-

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low it, nor injure the bridge by splintering away its corners. This thickness is made as small as possible, that the bridge may not be unnecessarily elevated, and the roadway is preserved nearly horizontal. The other courses of archstones too, do not often differ much in thickness from that at the crown. But although these things are pretty constant, there is a considerable degree of latitude in filling up the space between the back of the arch and the roadway. It may be done with substances varying in density, from the lightest charcoal or pumice, open shiver or chalk, to closely rammed clay, or even solid masonry; and it is not uncommon to make, in various ways, open spaces in the masonry of the spandrel, covering them above, so as still to support the roadway.

It will therefore be proper for us to enquire, what is the density requisite over every section of an arch, where the thickness of the crown is given, the roadway horizontal, the arch of uniform thickness, and the angles of abutment of the several sections constant, that is, all drawn from the same centre; or, what is the same thing, let us suppose the structure built up to the horizontal roadway with parallel sides, and then enquire, what is the proportion between the pressure borne by each section, in this way and the pressure of equilibrium; we shall thereby discover the ratio in which the density of the backing must, if needful, be diminished; and the quantity of expansion necessary towards the springing of the arch, that the advantages of equilibration may be preserved, even in this state of things.

Before we give a more rigid determination, we should wish to shew the practical builder, that the solution of this problem may be easily approximated to, by the help of the trigonometrical tables. For we may suppose the matter of the archstones to be the same in specific gravity with that which lies above it; and as there can be no impropriety in considering the arch as polygonal, from joint to joint, our mean versed sine is only half the sum of those at the two joints. The supposition is not strictly accurate, but it is sufficiently near: greater strictness would only serve to render the calculation more complicated, without making it more useful.

The following Table exhibits, in the first line, the supposed sections of the polygonal vault, taken 5° asunder. The second line is the angles of abutment, or inclination of each joint to the vertical. The third, the ratio of weight in each section, taking as the standard $872\frac{1}{2}$, or the length of the arch of 5° in 1000ths of radius; of course this line is merely the differences of the natural tangents taken at every 5°.

1	Key	2	3	4	5	6	7	8	9	10	11	12
2	$2\frac{1}{2}°$	$7\frac{1}{2}°$	$12\frac{1}{2}°$	$17\frac{1}{2}°$	$22\frac{1}{2}°$	$27\frac{1}{2}°$	$32\frac{1}{2}°$	$37\frac{1}{2}°$	$42\frac{1}{2}°$	$47\frac{1}{2}°$	$52\frac{1}{2}°$	$57\frac{1}{2}°$
3	872	880	900	936	989	1063	1164	1302	1489	1749	2119	2664
	The excess of each section over the key section will therefore be											
4		8	28	64	117	191	292	430	617	877	1247	1792
	But we must consider, that a part of the thickness at the crown is roadway, and this not, like the arch-stones, kept of uniform magnitude over each section, but must be diminished in the ratio of the differences of the sines of the abutments, or horizontal bases. These differences are											
5	872	869	859	842	819	790	755	714	668	617	561	499
	Now suppose the roadway, and other constant superstructure, to be at the crown of equal weight with the arch-stones; a supposition not far from the truth, and from which any small variation is not of great consequence; the weight of arch-stone and roadway will be a mean beween lines 3 and 5, or											
6	872	871	866	857	846	831	814	793	770	745	716	686
	And the excess of weight necessary on each section, will be											
7	0	9	34	79	143	232	350	509	719	1004	1403	1978
	Which is only about a fifth more than the preceding. And this must be provided for in the spandrel, or over the flank of the arch; the solidity of which will be found, by multiplying the numbers in line 5 as bases, into the mean height of the matter at each section. These mean heights, or versed sines, are											
8	.00024	.00381	.01519	.03407	.06037	.09369	.13397	.18085	.23306	.29289	.35721	.42642
	when the roadway is horizontal. And the effect of line 5, into line 8, which will express the end of the prismatic section over each arch-stone, will be											
9	.2	3.4	13.1	28.7	49.4	74.0	101.1	129.1	156.0	180.6	200.3	212.8
	In which observe, that the unit of height is radius, the bases being expressed in the same notation as in expressing the weights. If a number in this line be divided by its corresponding number in line 7, it will express the thickness at the crown, in terms of the radius, which equilibrates the matter over the corresponding section, between the arch and horizontal roadway; when that stuff is filled solid, and of equal density with the arch, of course these thicknesses are											
10		.39	.385	.363	.345	.319	.288	.254	.217	.180	.143	.108

The use of this will be understood by the following example. Suppose the thickness at crown $\frac{1}{7}$ of the radius of the circle, or .142857, which answers very nearly to the 11th key. We see by this last table, that the flanks of the arch over that key, or at 50° on each side of the arch, must be filled solid.

Theory. Between that and the crown they must be lightened, by using lighter matter, or making vacant spaces in the spandrel: and at a greater distance from the crown, the flank, although solid, will be too light for a crown of $\frac{1}{7}$; so that we must expand or increase the breadth of the arch, in order to preserve the equilibration. Every different thickness of crown will require a different arrangement in this respect. Without therefore prosecuting this farther, let us assume the thickness equal to $\frac{1}{10}$ of the radius, or $\frac{1}{20}$ of the span in a semicircle; a proportion not unusual, and of easy calculation; from thence, to find the density of the matter in the spandrel, take the numbers of line 7, divide by those of line 9, and multiply by the given thickness $\frac{1}{10}$, that is, divide by 10; we have for the density in the spandrel, in that case, Theory.

11		.26	.26	.270	.289	.313	.346	.394	.460	.555	.700	.93	(.13)

Accordingly, the density beginning at $\frac{1}{4}$ about the crown, must be about $\frac{1}{3}$ at 30° from it, and thence gradually increase till about 60°, where it must be 1, or equal to that of the arch. After that, if no denser material can be employed, the arch must be expanded in breadth, having already arrived at the limit of density.

If we make the thickness at the crown $\frac{1}{20}$ of radius, the densities will just be $\frac{1}{2}$ of the above numbers; the point of solidity will be removed a little farther from the crown; and indeed whatever be the thickness, the densities will be proportional to the above numbers, and may easily be had from them.

The above is for a horizontal roadway. There will be some alteration requisite if the road be made to slope up the arch: the quantity of pressure that is thus lost, must be corrected by increasing the density of the spandrel; and this will be more necessary towards the springing. It will not be difficult for the practical builder to form an idea of its effect. Take the section at any part, say 30° from the crown, where the horizontal distance is $\frac{1}{2}$ of radius; suppose the road to slope 1 in 10, for example, which is great, the fall will have become $\frac{1}{10}$ of radius, or .05; the versed sine is .1339, accordingly the height in the spandrel is reduced to .0839, and the density being increased, inversely as the height, we have .552 in this case at 30° instead of .346, other things being the same. Yet this density is too great; for the solid matter in the roadway will be increased, being lengthened by sloping. At the same time it admits of doubt, whether it may not be made thinner, in the same proportion; for its oblique position gives a greater vertical thickness. This will preserve the density at .552, and the whole series will be found, by deducting $\frac{1}{10}$ of the sine from the versed sine in col. 8, and proceeding with the remainders as with col. 8, as follows:

MVS	8	.00024	.00381	.01519	.03407	.06037	.09369	.13397	.18085	.23306	.29289	.35721	.42642
$\frac{1}{10}$ of MS		.218	.00872	.01736	.02588	.3420	.4226	.5000	.5735	.6428	.7071	.7660	.8192
diff.		—00194	—00491	—00217	+00819	.02611	.05143	.08397	.12349	.16968	.22218	.28061	.34450

And the effect of this height into the diff. of sines will be

	—1.6	—4.3	—1.8	+6.9	22.2	40.6	63.4	88.2	113.4	137.0	157.4	172.0
And the density	—2.2	—1.9	+1.15	.645	.57	.552	.577	.634	.733	.891	1.15	

Where we find the density in no case less than $\frac{1}{2}$, which is about 30° from the crown, and it increases both ways, about $\frac{3}{4}$ at 45° and at 19°, and solid about 53° and $16\frac{1}{2}$°, the first 10° are marked negative; for we should observe, that when we keep the thickness at the crown $=\frac{1}{10}$, the parallel to the roadway cuts the curve of arch-stones. We ought in fact to make the roadway of a proper thickness where the arch approaches nearest to it, and relieve the crown by rounding the two inclined planes into each other. This will also tend to diminish the density necessary in the spandrel; for the height will be a little increased, while at the same time a greater pressure is derived from the solid roadway. But we choose to allow the example to remain in this way, that the reader may see that every necessary information can be got, even in this way of considering it.

A smaller degree of slope, as 1 in 20, or 1 in 40, will tend to diminish still farther the density necessary in the spandrel, and approximate it to those found for the horizontal line. We might calculate the densities as well for these useful slopes, as for other thicknesses of the crown or proportions between the key-stones and superincumbent roadway, which, in the preceding enquiry, is taken at equality; but we forbear doing so, being satisfied with giving the intelligent practitioner clear ideas of the subject. He already knows there are pretty wide limits to his practice; and, if the case be any way delicate, we should think any person deserving the name of architect may, after what we have said, go over the necessary calculations for himself.

The mathematical reader will perhaps say, that we have taken a very awkward and unscientific mode of resolving this problem; we are not, however, inclined to admit that opinion. Our object has not been to give a specimen of the application of calculus; but to shew the practical builder how a good conception may be formed of the relative pressures in different parts of his arch, and this by a process purely arithmetical, and which is level to every capacity. We conceive that this is the way to make our speculations really useful, and perhaps it were well if scientific men had this oftener in view. Neither have we carried our results to many figures, like

Theory. some authors, who give five or six places of decimals; for we have considered that no common modes of measuring either distances, angles, or weights, can proceed to any thing near that nicety. Yet, that we may not rest satisfied with an approximation without shewing what degree of accuracy can be obtained, and especially that we may render this mode of conceiving the subject more useful by a more complete solution of the problem, we proceed to the following analytical investigation.

We have already shewn that the weights of the sections must be proportional to the differences of the tangents of the successive angles of abutment.

This is to be provided for,

1st, By the weight of the arch-stones; here taken as constant.

2d, By the weight of matter forming the roadway, &c.; here taken as of uniform thickness, and varying in effect only as the difference of the sines of the distances from the crown.

3d, By the matter in the spandrel; which may be made to vary in density, and is equal in the longitudinal section to the versed sine multiplied into the difference of the sines.

Take z, the angular distance from the vertex, δ the density in the spandrel, a the thickness of the arch or keystone, r the thickness of the road, &c. at the crown.

The sections are $=(a+r)$ flux. tan. $z=(a+r)(1+t^2z)\dot{z}$

The road is $=r\times$ flux. sine $z=r\dot{z}$ cos. z, wherefore

$(a+r)(1+\tan.^2 z)\dot{z}=a\dot{z}$ for the archstone

$+r$ cos. $z\dot{z}$ for the road

$+\delta\times$ cos. $z\dot{z}\times$ V sin. $\dot{z}$ for the spandrel

that is $a+r+t^2z(a+r)=a+r$ cos. $z+\delta$ cos $\dot{z}\times$ V sine z

and $t^2z(a+r)+r$ V sine $z=\delta$ cos. $z\times$ V sine z

$$\delta=\frac{\tan.^2}{\cos.\times \text{V sin.}}(a+r)+\frac{r}{\cos. z} \text{ or } \frac{S^2}{\cos.^3 \text{V sine}}(a+r)+\frac{r}{\cos.} \text{ or } \frac{\sec.\times\tan.^2}{\text{V sine}}(a+r)+r\times\sec. z$$

Which may be thus expressed:

In an arch of uniform thickness, with a horizontal roadway, given the thicknesses of the arch and roadway; required the density in every part of the spandrel, so that the whole may be preserved in equilibrio.

To twice the log. tangent of the angular distance from the vertex, add the log. secant. and subtract the log. versed sine; take the corresponding number, and multiply by the thickness of crown; and add to this the secant multiplied by the thickness of roadway. These, being expressed in terms of the radius, the resulting number gives the density in the spandrel, or proportion which the solid matter in measuring transversely across the arch bears to the whole breadth at the crown.

Accordingly, we have constructed the following short Table from this formula. The first line shews the multiplier for the thickness at the vertex. The second shews that of the roadway; and is merely the table of natural secants.

5°	10°	15°	20°	25°	30°	35°	40°	45°
2.01916	2.07809	2.18140	2.33757	2.560742	2.87293	3.309600	3.92952	4.82854
1.00382	1.01543	1.035276	1.064[illegible]	1.103378	1.15470	1.220775	1.305407	1.414214

50°	55°	60°	65°	70°	75°	80°	85°	90°
6.18556	8.513614	12.000	18.84713	33.54315	91.40552	224.144	1649.6946	Infin.
1.555724	1.743447	2.000	2.366202	2.923804	3.863703	5.75877	11.47371	Infin.

As an example of the use of this Table, let us take the thickness at the crown $=\frac{1}{10}$ of radius; and upon the supposition that the roadway, &c. is as thick as the archstones at the crown, we have, by multiplying as above directed, the following densities, viz.

at 5°	10°	15°	20°	25°	30°	35°	40°	45°
.25211	.25858	.26990	.28697	.31124	.34503	.39200	.45822	.55356

at 50°	55°	60°	65°	70°	75°	80°	85°
.69634	.93853	1.30000	2.00302	3.50051	9.33374	22.70234	165.5432

And if these densities be compared with those of line 11th, the reader will satisfy himself as to the value of the approximation which is there employed.

It would not be difficult, upon principles similar to the above, to establish a theorem for the elliptic, parabolic, and other curves similar to that we have now given fr the circle. But this is of less general use; and the limits assigned to an article of this kind, prevent us from entering at present upon an investigation, through which, perhaps, few of our readers would be inclined to follow us.

Another opportunity may be found of offering this to the public notice.

But, in the meantime, the reader must at once see, that by this mode of expressing the density in the spandrel, the solution we have given applies to any

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form of roadway. All that is necessary, is to compare the densities in the Table above given for a circular arch, with the relative height between the back of the archstone and the bottom of the roadway in the given design; and this will, we are sure, be more readily, and more satisfactorily done by the common builder with his sector and compasses, than by giving him equations for any number of figures of extrados.

We have arrived at a theory which is very nearly consonant with what has become, among our modern builders, the most approved way of distributing the weight over the haunches of an arch. It was customary, in the construction of bridges, to fill up the haunch with solid matter, such as gravel, earth, or the like, until a roadway of a proper slope was procured. Where the arches were small, this might not be attended with any perceptible bad effect, provided the archstones were of a good depth. But the necessity of lightening the haunches, has been forced upon the attention of builders, whenever large arches have been attempted. A more remarkable instance of this we cannot have than in the bridge of Pontypridd, in Wales, built by William Edwards, a simple country mason of Glamorganshire. It is one of the boldest arches in Britain, perhaps even in Europe, being 140 feet span, and 35 feet rise; a portion of a circle of 87½ feet radius, and the depth of archstones is only 3 feet.

In each haunch there are three cylindrical openings running through from side to side: the diameter of the lowest is nine feet, of the next six feet, and of the uppermost three feet; and the width of the bridge is about eleven feet. To strengthen it horizontally, it is made widest at the abutment, from whence it contracts towards the centre, in the old and unartificial way, by seven offsets, so that the roadway is one foot nine inches wider at the extremities than at the middle; and is also very steep.

That this mode of lightening the haunches is effectual, we have undoubted proof in the case before us. It is not ungraceful, but were it thought so, the tunnels might be concealed by the side walls. Indeed these tunnels might even become useful, by affording additional passage for the waters in dangerous floods, as seems to have been intended in the ancient bridge of Merida, the Pont St Esprit over the Rhone, and in many other similar structures both ancient and modern. Nevertheless we cannot approve of this mode. It seems to press unequally on the arch, and only at a number of detached points; and though the widest tunnel may be placed just where the greatest evacuation is necessary, yet this ill agrees with the gradual approximation to solidity, which we should find in passing down the back of the arch. To make correct workmanship in these tunnels is troublesome and expensive. The following mode, which has now become the customary practice of our most experienced bridge builders, is much preferable.

Plate LXXXIII.

Thin longitudinal walls are built over the flank of the arch parallel to the sides of the bridge, and about three or four feet asunder. The spaces between are covered at top with thin flat stones, or arched over by pointed or circular arches; or they are covered by regularly projecting courses in the way of an Egyptian arch; and in any case a platform is thus formed, upon which the gravel may be laid for a roadway.

In all probability, the first inventors of this mode of building, besides employing it with the view of equilibrating the arch by lightening the part over the haunches, had also an idea of steadying it by the lateral abutment. They appear to have considered these spandrel walls as a sort of hoops, that would keep the parts of the arch together, and hinder any stone from moving, by their great friction, inertia, and mutual abutment. Hence various ingenious modes have been employed for locking them into the back of the archstones, propagating the pressure through, and securing them from sliding away at the bases.

They indeed act in this way; nevertheless the equilibration of the arch should be attended to in their construction, that every unnecessary strain may be avoided. The thickness of these walls may be varied indefinitely, and the vacant spaces made in any proportion to the solid parts. The walls ought to be near each other, that their effect may be felt over the whole arch, and perhaps they should spread out towards the bottom; but this is not so very necessary, for the courses of archstones *break joint* with each other, and the inequality of pressure in one course is immediately corrected by being propogated to the succeeding. We may determine readily the thickness proper for these walls, by the help of the Table last given, provided we know the thickness of the arch, and of the roadway, (including the small arcade below it,) and the breadth of the whole structure. For example, let the breadth of the soffit be 20 feet, the thickness at crown $\frac{1}{10}$ of the radius, and the archstone alone $\frac{1}{20}$; being the same proportions as for the numbers in the example to last Table. Then from that example we find the thickness of all the masonry in the spandrel must, near the crown, be $\frac{1}{4}$, or 5 feet; at 30° from the crown, it must be 7 feet; at 40°, 9 feet 4½ inches; at 50°, 13 feet 11 inches; and at 57° or 58°, the whole must be solid masonry. Suppose, in the next place, that the side walls are 18 inches thick, and the spandrels 3 in number, of course there will be four openings. The thickness of each wall must be at 50° from the crown 3 feet 8 inches; at 40°, 2 feet 1½ inches; at 30°, 1 foot 4 inches; and diminishing from thence to half that thickness. Perhaps eighteen inches is too thin for the side walls, but they may be thickened towards their bases, diminishing the thickness of the spandrels in proportion. On the other hand, nine inches appears too little for the spandrel wall, when we consider that an arch is to be built on it; but the height near the crown will be so small, that a little additional thickness there will be of no moment; nay, it will enable the arch the better to resist any overload at the crown.

Equilibration of semicircular arches.

We have now determined a method of constructing an equilibrated arch for sixty degrees on each side of the vertex; and this method, so far from having any thing unusual, is even strictly analogous to that which is adopted by the practical builder. Why then cannot we keep pace with him throughout, and give a construction for the entire semicircle? No difficulty is felt by the mason in that case. He constructs

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qual to the resistance of any pressure that is likely to be opposed to it. And when the arch is a segment much smaller than a semicircle, the rules we have already given for its equilibration must be considered. But, instead of solid courses of masonry, the haunches of arches are often filled up with coarse gravel or shiver, and sometimes with mere earth or sand. Materials of this description does by no means act by mere dead weight. It has a tendency to slide down towards a horizontal position; and, of course, possesses, in some slight degree, the *quaquaversum* pressure of a fluid. This may act on our arch in a manner altogether new, and produce strains for which hitherto we have made no provision. We shall first consider the back of the arch as filled up with a fluid substance, as water. The pressure in every part will be in a direction perpendicular to the curve, and will be proportional to the depth. A pressure perpendicular to the curve will be equivalent, in effect, to a vertical pressure, which exceeds it in the ratio of the secant of the inclination to the vertical. Of course, the pressure at the springing, when all is equilibrated, must be equal to the horizontal thrust in a semicircular arch. Take the thickness of matter at the crown $=\frac{1}{m}$ of radius, the weight of one degree $=k$, then the horizontal thrust will be $57\frac{1}{7}k$, and the height of fluid necessary for this will be $57\frac{1}{7}$ times the thickness at vertex, provided the specific gravity of the fluid be the same with that of the arch. But if not, let $f=$ the gravity of the fluid, and $S=$ that of the arch at vertex, then $\frac{57\frac{1}{7}\,S\,R}{f\,m}$ will be the height required. Suppose the arch made of brick, which is about double the specific gravity of water; and we have, for water filling up the flanks, till just covering the crown of the arch, a depth at the springing nearly equal to the radius; and, of course, the thickness at crown should be about $\frac{1}{114}$ R, or $\frac{1}{228}$ of the span, when in equilibration at the springing. We take no notice of the effect of the arch in assisting this. Water, therefore, is much too light for equilibrating an arch at the springing, in any moderate thickness of crown. It might, however, be so employed. The quantity requisite is always finite, even at the vertical spring courses; and by expanding the arch, or otherwise employing its hydrostatical properties, the requisite weight of fluid could without doubt be obtained in any case. But it is unnecessary to pursue this speculation farther than merely to observe, that its weight on the arch, where a variation is requisite, might be adjusted, by attending to the modes of altering the density which we have noticed, when speaking of filling up the arch by masonry alone.

Though the action of sand, gravel, or mould, in situations such as this, be not exactly the same with that of water, in following the laws of hydrostatical pressure; yet these materials resemble water, and may be conceived to hold the middle place between the fluid and the solid backing. In some respects they are more advantageous than the fluid. They are stiffer, so to speak, affording a lateral abutment to the arch, if it is likely to yield; and as the parts have a great friction among themselves, it will require a much greater pressure acting horizontally, to make the matter rise, than in the case of a fluid. We must not, however, be too confident. Materials of this kind are compressible; and we have already seen, that very slight shifts are attended with dangerous consequences. At the same time, we need not be much afraid of a trivial departure from exact equilibration; for it is not likely that materials of this kind will act with the powerful effort of hydrostatical pressure.

But there is another case, where matter of this kind is likely to be attended with more pernicious effects than even a fluid of equal density would be. We mean, when the back of the arch is gorged up with water from land floods, if the backing be open gravel, or shiver, we have superadded to its weight that of the whole quantity of water admitted into the structure. This, even if it acts equally on both sides, must be a dangerous experiment on any arch; but where it is confined to one side, as is generally the case, and between lofty side walls, the effects are likely to be serious indeed. Accordingly, the builder forms gutters in the side wall to let off the water ere it collect. A practice which is in general highly useful; but which, in the case of sand, clay, or mould, is of small service. The water enters into such matter by its capillary attraction; and fills it to the upper surface in spite of our gutters. It of course expands it, and this with a force which we cannot measure, but which we are sure is very great. Here the friction of the parts, which was so useful in the former instance, proves extremely hurtful. For as the matter cannot easily rise, and probably the adhesion of its particles is increased by the water, the expanding force becomes an enormous hydrostatical pressure acting perpendicularly on the side walls and extrados of our arch, and which in all probability they may not sustain.

We do not mean to pursue the theory of the pressures exerted by these semifluids any further at present. We look upon their use in this case as radically bad, and would recommend its discontinuance. If the reader wishes for more information on the subject, he will find it when we come to speak of retaining *walls*. In the meantime we may remark, that the dangerous consequences of this mode of backing are, in some degree, prevented by ramming the layers of matter, especially if it consists of mould or the like; or, by puddling them so as to form a mass impervious to water. And here we should observe, that as this ramming will produce an extraordinary lateral pressure, we must attend to equilibration, as we rise along the arch, and secure the side walls, by thickening them below, or curving them horizontally or vertically.

On the thickness of the archstones.

The thickness of the archstones is an important department of the theory of arches. It is natural that we should endeavour to make them as small as possible. That will diminish the expense of the structure, lessen the pressures in the arch, and increase the security at the springing. But there is an evident limit to this diminution; for though we take every pains to render the joints close, the stones may come at length to be so small as to crush by the thrust of the arch. This is, indeed, a curious branch

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The question evidently depends on the amount of the tangential pressure. At the crown this is the horizontal thrust. We shall suppose all the joints to be duly drawn to equilibration, the sections fairly abutting on each other, and no weakness arising from acute angles.

Stone, it is said, will carry from 250,000 to 850,000 lb avoirdupois per foot square, and brick 300,000 lb. They have been made practically to carry $\frac{1}{8}$ of this, and even more. The pillar in the centre of the Chapter House at Elgin carries upwards of 40,000 lb on the square foot, and there was formerly a heavy lead roof on it. It is a red sand stone, and has borne this pressure for centuries.

We shall therefore take 50,000 lb per foot as a load, which may be safely laid on every square foot in the arch. A cubic foot of stone weighs about 160 lb per foot; and brick weighs less. Suppose, therefore, the arch to be one foot thick at the crown, and the keystone one cubic foot, it will bear a horizontal thrust of 50,000 lb, that is, $312\frac{1}{2}$ times its weight.

But, 50,000 : 160 :: R : Tang. 11′ 0″ 3‴, which will be the angle of the key-stone in that case. So that an arch of $312\frac{1}{2}$ feet radius, or a semicircular arch of 625 feet span, might bear to have a key-stone of a foot deep, without risking its being crushed more than in structures which have already stood for many years. And this may be called the limit of stone arch building; for if we double the depth of the stone, we will thereby double the weight also, and its ratio to the horizontal thrust will still be the same. Indeed this limit does not much exceed what has been actually executed. A considerable portion of the bridge of Neuilly is an arch of 250 feet radius; and Gautier mentions a platband in the church of the Jesuits at Nismes, the camber of which, after settling, would make it a portion of an arch of 280 feet radius. The length or span is $26\frac{1}{2}$ French feet, the rise only 4 inches, and therefore the diameter of its circle would be 560 English feet.

This singularly bold platband was made under the conduct of Pere Mourgues, after the design of Cubisol, an able architect. The stones are 1 foot thick, their depth is 2 feet towards the key, and 2 feet 4 inches at each end. It had a camber given it of about 6 or 7 inches, and descended near 3 inches on striking the centres. (*Gautier.*)

We see, that the horizontal pressure does not determine the vertical thickness of the arch-stone. But as we pass down the arch, it is plain that the butting surfaces must increase, in proportion to the increasing tangential pressure.

At sixty degrees from the vertex, granting that the arch is equilibrated, the depth of the arch-stones must be doubled; and though the equilibration be carried no farther, yet, at the springing or horizontal joint, a small increase will still be necessary. The ratio will soon be found. To the square of the weight of the semi-arch, add the square of the horizontal thrust, the square root of the sum is the pressure at the springing. If we divide this by the horizontal thrust, it will give the thickness at the springing, compared with that which is necessary at the crown. Or if we divide it by $312\frac{1}{2}$, it will give the smallest depth of joint which should be used at the springing. The thrust and weight are supposed to be given in solid feet. If given in pounds, divide the above quotient by 160, or divide at once by 50,000.

Example. Required the thickness of the lower joints for a semicircular arch, when the weight of a section of a foot in breadth from the crown of the arch to the springing is 60,000 lb, and the horizontal thrust is 20,000 lb, which answers nearly to a 60 feet arch, 4 feet thick at the crown.

```
 60          20      50,000)63,250(
 60          20             1,265 feet,
3600         400
 400
4000(63.25
 36
123)400
    369
     310 &c.
```

or 1 foot $3\frac{1}{5}$ inches nearly; of course the vertical section, or key stone, might be only $\frac{20000}{50000}=\frac{2}{5}$ of a foot, or $4\frac{4}{5}$ inches, if it were necessary so to reduce it.

For another example, take a 50 feet arch, having 5 feet thickness at crown. The semi-arch may be found sufficiently near, by multiplying the half span into the half height to the road, viz. $25\times15=375$. And the horizontal thrust $5\times25=125$ feet of stone, $375^2=140600$, $125^2=13625$, their sum is 156250, the square root of which, divided by $312\frac{1}{2}$, gives 1.265, or 1 foot 3 inches; and here again the vertical section might be $\frac{125}{312\frac{1}{2}}$, or $4\frac{4}{5}$ inches only.

If we calculate upon the same principles, the depth of arch stone at the spring course of a semi-circle of 100 feet span, 10 feet thick at crown, we shall find it to be 5 feet, and at the crown the depth may be 19 inches. In the great arches of the bridge of Neuilly, the thickness at the crown is about 4 feet 8 inches, the span 128.2 feet, and height 32. The horizontal thrust is great, the crown being drawn with a radius of 150 feet; consequently this arch would require a depth at springing of about 4 feet. But when the centre was struck, the crown of this arch descended 23 inches, which has rendered it a portion of a much larger circle, and has greatly increased the horizontal thrust. After all, the pressure at the springing is scarcely greater than in the last example, and the depth of joint there need not have exceeded 5 feet. It is nearly three times that, and even at the crown the thickness is $\frac{1}{4}$ greater than the increased thrust would require. We trust, therefore, that, in spite of the great risk this singular arch has run, it may yet long remain a monument of the skill and boldness of the able architect who designed it.

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It may be proper to observe, that the French architects Perronet and Soufflot, made an experiment on the strength of the stone of which it was composed. They found, that a cubic foot of it, which weighs 152 ℔, required 240,000 ℔. to crush it. In the above investigation we have only taken it at 50,000.

The thickness at the crown of the arch, cannot, with propriety, be reduced so much as we have supposed in the above examples. This part of the structure is liable to be strained transversely. And it has been found, that when stone, or other matter, is bearing a great pressure longitudinally, its strength against a transverse strain is thereby much diminished. But, independent of that, there is another cause for preserving the crown of a greater thickness. The varying pressure of carriages would be apt to produce some motion among small stones; this would chip away their angles, and accelerate the destruction of the building. But there is seldom any need for this reduction. In most cases, it would only be additional labour.

Of Piers.

Of Piers.

The piers and abutments of a bridge must be so constructed, that each arch may stand independent of its neighbours. For though, by the mutual abutment of arch against arch, the whole may rest upon very slender piers, if once the structure is erected; yet, as they must be formed singly, and are exposed to many accidents, it will be best to contrive them, that the destruction of one arch may not involve in it that of the whole.

Some of the writers, on the principles of bridges, in treating this department of their subject, have found it necessary, by the help of the higher calculus, to find the centre of gravity of the semi-arch. The solution of the problem, we are convinced, so far as it is useful in practice, lies much nearer the surface.

The reader has already frequently seen, that the ultimate pressure may, in every case, be reduced to two others, viz. the weight of the semi-arch above, and the horizontal thrust. In the equilibrated arch, this pressure is directed perpendicularly to the joints of the sections; and these being usually drawn at right angles to the curve, the pressure is in the direction of the tangent to the arch. Hence, we have often called it the tangential pressure. Upon this principle, however, when the curve springs at right angles to the horizon, an infinite pressure is required in the vertical direction,—a supposition which cannot have place in practice. We must accordingly call in the assistance of friction in that case; a force which may be set in opposition to the horizontal thrust, and which, increasing with the superincumbent weight, very fortunately keeps pace also with what it is intended to oppose.

Granting, then, that the friction is so contrived, upon the principles already explained, that there is no danger of any slide at the horizontal or springing joint; it will be readily admitted, that no slide is likely to take place in any horizontal course below that, till we arrive at the foundation; for the disturbing force is constant, but the friction increases as we descend. Our principal care then must be, that the pier does not overset, by turning on the farther joint E of its base, as a fulcrum. Take a in the horizontal joint, Aa as the centre of pressure. Draw aV to represent the weight of the semi-arch, and VT the horizontal thrust; then Ta is the ultimate pressure: and if, when produced, it falls within the base of the pier, it is perfectly obvious that it can never overturn it. And this is altogether independent of the weight of the pier; for if that were a mass of ice, immersed to the springing in water, the case would be exactly the same.

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Plate LXXXI. Fig. 6.

But the pier itself has a considerable stability, arising from its own weight; and even though the direction of the ultimate pressure of the arch alone pass out of the base, the tendency to overturn the pier may be balanced by its weight. This weight may be supposed concentrated in the centre of gravity of the pier, and of course to act in the vertical line which bisects it.

Its effect will be nearly found by laying off in that line from the point q, where the direction of the ultimate pressure of the arch intersects it, qr=to the weight of the pier, and taking qs=the ultimate pressure=aT, and completing the parallelogram, the diagonal drawn from q will represent the direction and magnitude of the united pressure of the arch and pier. This is not strictly accurate; it would be so if a and q coincided, which is the case with a single arch standing on a pillar: but in general, the ultimate pressure is still more favourable than this. Its direction at any point is in the tangent of a curve, which approaches the vertical as we descend, since the proportion arising from the weight of the pier increases with its height.

In order to find analytical expressions for these forces, let the horizontal thrust of the arch $=t$. The weight of the half arch $=a$, and that of the pier $=p$, the height of the pier to the springing of the arch $=h$, the breadth at the base $=b$.

1. Then the horizontal thrust acting in AG, tends to overturn the pier, and its force round the fulcrum E will be represented by multiplying it by the perpendicular distance AD= viz. $h \times t$.

2. The weight of the pier acts in the direction BC, and its effect will be represented by multiplying it by the leverage CE, viz. $p \times \frac{1}{2} b$.

3. The arch acts with the leverage EK, which is not equal to the breadth of the pier, by the part KD=AH, say $\frac{1}{2}$ of the depth of the joint at the springing. This will never exceed one-fourth of the breadth, when two different rings of arch-stones rise from the same pier, unless the pier widen below. Call EK, therefore $=\frac{3}{4} b$.

We have now $ht = \frac{1}{2} bp + \frac{3}{4} ba$; whence,

1st, $b = \frac{ht}{\frac{1}{2}p + \frac{3}{4}a} = \frac{4ht}{2p + 3a}$, and consequently,

To find the least breadth of the pier at its base, divide the horizontal thrust by half the pier added to three fourths of the half arch. Multiply the height of the pier by the quotient.

2d, $h = \frac{b(\frac{1}{2}p + \frac{3}{4}a)}{t}$, that is,

The height of a pier to the springing, having a

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given base and weight, is found by adding the half pier to three fourths of the arch, multiplying by the breadth of the base, and dividing by the horizontal thrust.

$$3d,\ p=\frac{h\,t-\frac{3}{4}b\,a}{\frac{1}{2}b}=\frac{2\,h\,t}{b}-1\tfrac{1}{2}a\,;$$

or the weight of the pier cannot be less than the excess of the horizontal thrust multiplied by twice the height of the pier, and divided by the base, above one and a half times the semi-arch.

In the above determination it may be observed, that we consider the weight of the pier as independent of its base. Now, though it may be said with propriety, that the weight of the pier cannot be known until we know its thickness, which is the very thing sought, yet a little consideration will shew, that we may give different magnitudes to piers which have equal bases, and that, either by altering the outline of their sides, the density of their structure, the gravity of their materials, or the weight of solid matter over them, we may therefore, when the base is given, apply the weight necessary to keep the pier in equilibrio, provided this does not require the pier to be any more than a solid mass up to the roadway. Should the base assumed admit of the pier being much less than the solid parallelopiped, we may diminish it in various ways; as, 1st, By opening arches over the pier, where, in case of floods, we will procure an addition to the water-way; a practice very usual in the ancient structures: or, 2d, By tapering the pier towards the springing of the arches, or by making each pier only a row of pillars in the line of the stream, arching them together at top; a mode which may perhaps be objectionable in a water-way, but which would have a very striking and light effect in land arches. Something of this kind has been done by Perronet at the Pont St Maxence.

When piers indeed are to be exceedingly high, as in the columns which are sometimes employed in supporting a lofty aqueduct, the best way is to make them hollow, and give them stability, by enlarging the base. They will, in that case, press less on the foundations, be less expensive, and they may be greatly stiffened by hooping.

Indeed it is not usual to make piers solid all the way up to the road; the spandrel-walls are carried back so far as to unite with those of the neighbouring arch, are locked together by a cross wall just over the middle of the pier, having also walls longitudinally, and the whole arched or flagged over from spandrel to spandrel just under the roadways.

Nevertheless, as the case of solidity will enable us to assign a limit to the breadth of piers, which it may be proper to be acquainted with, we shall proceed in that investigation.

The weight of the pier in that case will be as the rectangle under its height and thickness, expressing the weight of arch and pier by the cubic feet of stone. The pier indeed will be somewhat more; for the sterlings or breakwaters, at each end, will add something to its stability; and this will be still further increased in proportion to the horizontal push, if the whole bridge be wider at the foundation than at top, as is very common. Excluding these collateral advantages, we shall consider the whole as rectangular, and then the stability may be found in the longitudinal section. We have already $b=\frac{h\,t}{\frac{1}{2}p+\frac{3}{4}a}$, and in the case of a parallelogram $\frac{1}{2}p=\frac{1}{2}b(h+c)$, c being the height from springing to the roadway. By substitution there arises $\frac{1}{2}b^2(h+c)+\frac{3}{4}a\,b=h\,t$; and by resolving this quadratic equation, we have

$$b=\sqrt{\frac{2\,h\,t}{h+c}+\left(\frac{3\,a}{4(h+c)}\right)^2}-\frac{3\,a}{4(h+c)}$$

or thus, $b=\frac{\sqrt{2(h+c)\,h\,t+\frac{3}{4}a|^2}-\frac{3}{4}a}{h+c}$ as a formula for the thickness of solid piers to support equilibrated arches; and it must be observed, that if the arch be understood to act otherwise than at $\frac{3}{4}$ the thickness of the pier, this coefficient may be altered accordingly.

Explanation of the formulæ.

As an example of the use of the above, take an arch of 100 feet span, six feet thick at the crown and semicircular. The horizontal thrust is $6\times50=300$ cubic feet; and let us take the weight of the half arch as $=1200$ at a medium, since, on account of the open spandrel, it may be considerably varied. Suppose the arch sprung at 18 feet high, then $h+c=74$

$$\frac{2\,h\,t}{h+c}=\frac{2.\ 18.\ 300}{74}=146,$$

also $\overline{\frac{3.1200}{4.74}}|^2=\left(\frac{3\,a}{4(h+c)}\right)^2=147.93$,

and $\sqrt{146+147.93}=17.14$,

from which subtract $\frac{3\,a}{4(h+c)}=12.17$, we have 4.97, or 5 feet nearly, for the thickness of the pier, which is not one-twentieth of the span. In an example nearly the same as this, 13 feet has been given by an eminent mathematician for the thickness of the pier; but the reason is, that the stability which the pier derives from the superincumbent arch, has not been taken into consideration; an oversight the more extraordinary, since it is evident, that unless this weight did bear completely on the pier, it could have no tendency whatever to overturn it.

Suppose that c in the above formula is $=0$, or, what is the same thing, that the pier is carried no higher than the springing,

we have $b=\sqrt{2\,t+\overline{\frac{3\,a}{4\,h}}|^2}-\frac{3\,a}{4\,h}$

And in an arch of the above dimensions,

$2t=600$, $\frac{3\,a}{4\,h}=\frac{3\times1200}{4\times18}=50$, when squared $=2500$

$\sqrt{3100}-50=55.68-50=5.68$ nearly, or about a seventh part more than the former. We see therefore how little the stability may depend on the mere weight of the pier.

We may have a proof of the accuracy of this determination, by comparing it with the formula first given for the thickness of piers, viz. $b=\frac{h\,t}{\frac{1}{2}p+\frac{3}{4}a}$, $h\,t$, or the overturning force, will be $300\times18=5400$.

Theory. The pier in the first case, taking it at 5 feet, will be $5 \times 74 = 370$, and $\frac{1}{2}p + \frac{3}{4}a$ will be $185 + 900$ or 1085; multiply this by 5, we have 5425, a little more only than the overturning force, as the thickness was taken at 5 feet, which is a little in excess. The reader, if he chooses to go through the calculation for himself, will find 4.97 agree exactly.

In the second case, the pier $= 5.68$ nearly, $\times 18 = 102.24$, and its half $= 51.12$, which added to 900, and multiplied by 5.68, gives 5402.3. A trifle in excess, because 5.68, like the former, is only an approximate number.

The weight of the pier in this case making so small a part of the whole resisting force, we may readily believe, that its total immersion in water would make no great addition to the requisite thickness. Stone, when so immersed, loses about $\frac{2}{5}$ of its weight, being in specific gravity about $2\frac{1}{2}$ times that of water; and, in the above example, were the whole pier under water, it ought to be about a fiftieth part thicker.

We have hitherto supposed the arch equilibrated, at least as far as is conveniently practicable, in which case the horizontal thrust is represented by the rectangle under the radius and thickness at crown. But if the equilibration of the arch has not been attended to, we must consider whether any uncommon weight about the shoulders may not produce, by the help of friction, a thrust in the arch fully equivalent to what would arise from a greater thickness at the crown; and our calculations are to be regulated accordingly.

On the other hand, we have given the arch a weight in the above example which is nearly that of solidity. But in general the arch weighs much less. The most common case, where the stability of the pier is any way doubtful, is when it carries no more than the ring of arch-stones, and before it is assisted by the weight of the superincumbent backing. The weight keeping the pier steady, is now much diminished; while the horizontal thrust is unaltered; for, if not propagated by weight, it is by means of the friction of the sections propagated to the pier, so as to act against it in the same manner as if completed.

Now, as it is by no means likely that the arch will be made thinner at the spring-courses than at the crown, while any additional thickness of the former is always in favour of the piers, we shall proceed upon the supposition, that a regular annulus, or ring of stones, is laid on them everywhere of equal thickness. Suppose this thickness, as before, to be 6 feet. In that case the semi-arch of the above dimensions measures 499.5, or 500 feet, and $\sqrt{2t + \left(\frac{3a}{4h}\right)^2} - \frac{3a}{4h} = \sqrt{600 + \overline{\frac{1500}{4.18}}|^2} - \frac{1500}{4.18} = 32.13 - 20.83$, or $11\frac{3}{10}$ feet for the breadth of the pier. But it is by no means likely that the arch would have 6 feet thickness of crown in these circumstances; 2, or at most 3 feet, would, in all probability be thought sufficient for a depth of keystone; and a ring of arch-stones 2 feet deep will require a pier of 9 feet only. If we build up the pier behind the springing for about 6 feet, this thickness may be reduced to 8 feet; and it will be absolutely necessary to do so in a case of this kind, to prevent the lower sections of the arch from sliding away.

The above example is taken for a semicircular arch; and though the reader must see, that the thickness of the pier is in no certain proportion to the span, it is nevertheless obvious, that those writers who derive it from that, have hitherto erred considerably in excess. It is usually stated at $\frac{1}{6}$ for semicircles; but we see, that in the most unfavourable circumstances, it need not exceed $\frac{1}{9}$ of the span, and may often be made much less. This, however, we state with limitation, referring to the height of pier above given; for were the pier much higher, it must be made thicker; if the pier be infinitely high, the weight of the arch sinks into insignificance, and the thickness, $= \sqrt{2t}$, which in the above arch 6 feet thick is $= 24\frac{1}{2}$ feet nearly, and in general, if the thickness at crown $= \frac{1}{n}$ of radius, then $\sqrt{2t} = \sqrt{\frac{2r^2}{n}} = r\sqrt{\frac{2}{n}}$, that is, taking s the span, $m = 2n$, the thickness $= s\sqrt{\frac{1}{m}}$; whence this rule for the thickness of a pier of infinite height. Find what part the thickness at crown is of the span, extract the square root, and multiply it by the span for the thickness; or thus, multiply the diameter by the thickness at crown, and extract the square root.

One of the loftiest bridges with which we are acquainted is that of Alcantara, over the Tagus, in Spain. It is stated by Don Antonio Ponz, in his *Viage d' Espana*, to consist of six arches, the two largest 110 feet in span, the water at the lowest is 42 feet deep; from the surface of which, to the beginning of the springing of the middle arches, 87; and from thence to the upper surface, 76; which, with the 4 feet and a half of parapet, make the whole 205 feet and a half, (more correctly, $209\frac{1}{2}$). Taking then the thickness at crown as equivalent to 16 feet, and the diameter 110, the thickness for an infinite height should be 42 feet. They are 38 in thickness, and 129 feet high. Let us now try this thickness by the general formula given in the earlier part of this Section.

The lower or immersed part 42 feet high, and 38 broad, is 1596; but of this $\frac{2}{5}$ are to be deducted on account of the immersion, leaving for that part 958. The pier from thence to the springing is 87 by 38, or 3306. We must suppose such a pier built up between the arches to at least $\frac{1}{3}$ of the height, or about 20 feet; but on account of a set off which appears in the design, we shall suppose the breadth still 38 on an average, which makes 760, and the whole pier 5024, and its half is 2512. To this add $\frac{3}{4}$ of the semi-arch; say $\frac{3}{4} \times 55 \times 16 = 660$, and we have 3172. By this number let us divide the product of the horizontal thrust and height of pier, that is, $16 \times 55 \times 129 = 113520$, and we find about 36 feet, very near

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The Rise of Water produced by Obstructions to the Current, &c.—Continued.

Velocity.		Description of River.		Obstructions.									
Per Sec.	P. Hour.	The Current usually termed	The Bottom, which just bears such velocities	$\frac{1}{3}$		$\frac{1}{2}$		$\frac{5}{8}$		$\frac{3}{4}$		$\frac{7}{8}$	
				Head of water, and velocity produced at the obstruction in feet.									
Ft. In.	Miles.			Head.	Vel.	Head.	Vel.	Head.	Vel.	Head.	Vel.	Head.	Vel.
$\frac{1}{4}$ or 3	$\frac{1}{6}$	Dull	Ouse and mud	.0014	.394	.0033	.52	.0067	.7	.0162	1.05	.068	2.1
$\frac{1}{2}$ or 6	$\frac{1}{3}$	Gliding	Soft clay	.0058	.787	.0133	1.05	.0267	1.4	.0647	2.1	.274	4.2
I.	$\frac{15}{22}$ or $\frac{2}{3}$	Smooth	Sand	.0231	1.575	.0532	2.1	.1069	2.8	.259	4.2	1.086	8.4
II.	$1\frac{4}{3}$	Uniform tenors	Gravel	.0924	2.75	.2128	4.2	.4276	5.6	1.036	8.4	4.344	16.8
III.	$2\frac{1}{22}$	Ordinary Freshes	Pebbles	.2079	4.325	.4788	6.3	.9621	8.4	2.331	12.6	9.774	25.2
IV.	$2\frac{8}{11}$		Shivers and chingle	.3696	5.5	.8412	8.4	1.7104	11.2	4.144	16.8	17.376	33.6
V.	$3\frac{9}{22}$	Extraordinary Floods and rapids	Boulders and soft schistus	.5775	7.875	1.3200	10.5	2.6725	14.0	6.475	21.0	27.150	42.
VI.	$4\frac{1}{11}$		Stratified rocks	.8316	9.45	1.9152	12.6	3.8484	16.8	9.324	25.2	39.096	50.4
X.	$6\frac{9}{11}$	Torrents and cataracts	Indurated rocks	2.3100	15.75	5.320	21.	10.69	28.	25.9	42.	108.6	84.

By the help of this Table, we may see the effects likely to be produced in rivers by the usual accidents to which they are liable. The velocities above ten feet produce inundations that sweep away every kind of structure. Those in the latter part of the Table are given as fair results of the theory, but, in fact, they are impracticable.

In Westminster Bridge, the piers form about one-sixth of the water-way; the velocity is between 2 and 3 feet, or more accurately $2\frac{1}{4}$ feet; the head, therefore, will be between .036 and .082, more accurately .045, or about half an inch; which is exactly the greatest fall observed by Labelye.

At London Bridge, the apparent water-way is only one-fourth of the breadth of the river, but is much reduced by the drip shot piles, which have been driven into the bed to protect the foundations. The velocity of the stream above the bridge is 3 feet 2 inches, which, by this Table, would give a head of 2.6 feet, and by the former one 4 feet. We cannot suppose these piles to take off less than one-fifth of the water-way, which would make the head 4.31 by this Table. But probably the contraction is greater than this Table supposes, coming nearer that assumed in the former, (which would have given us a head of almost 6 feet,) since a fall of 4 feet 9 inches was observed about the year 1730; and the excavation had become so very dangerous, as to suggest the measure of cutting out one of the piers of the bridge, and throwing two arches into one.

The fall at Blackfriars will be somewhat less than at Westminster, but will not exceed one inch. In the same example, as before tried, this Table affords the following results:

1st, *The Bridge in common Waters.*

Original depth	3	4
Obstruction $\frac{1}{3}$	1	1.33
	4	5.3
Contraction $\frac{1}{20}$	.2	.26
	4.2	5.6
Original depth	3	4
Depth cut by the river	1.2	1.6

So that the piers are safe in common waters.

2d, *The Bridge in Floods.*

General depth	6
Add for obstruction $\frac{5}{8}$, or	3.75
	9.75
Contraction $\frac{1}{20}$	.487
Depth when general velocity restored, viz. 5 feet per second	9.137
Add $\frac{2}{3}$ to bring it to tenor velocity	6.092
The pebbly stratum will be cut until the depth is	15.23
But there is only	9.
There will be cut below the bed	6.23

Now, since the pebbles extend only to three feet, the waters pass to the clay, which bears only one-third of this velocity, and would therefore require a depth of 45.69, or 36.69 below the bed. The bridge therefore cannot stand in such floods as this. Suppose, then, that it be proposed to make a total change of foundation, as, by paving all across the river, or any similar operation, referring to the Table with an obstruction of $\frac{5}{8}$, and velocity 5 feet, we find the

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head produced is 2.6725, and velocity 14 feet per second, which would require a bottom as firm as solid rock. With good workmanship, however, the pavement would stand a considerable time, especially if the joints were so carefully closed that water could not readily penetrate, and work out the finer materials in which the pavement was bedded: For, although the water passes through the arch with this great rapidity, yet the general river being in a different train, and running with a much smaller velocity, will not bring along with it much heavier materials than the gravel and pebbles of the bed, and these will not be very injurious to the artificial bed at the bridge: For we are of opinion, that it is by no means the action of the water, but rather the attrition, or battering and rubbing of the boulderstones, gravel, and sand, brought down by it, that renders the hardest rock liable to be cut up by the force of a swift running stream. It is, nevertheless, extremely difficult so to secure a pavement, or inverted arch, in a river, that the water will not ultimately carry it away, even when the river does not run foul in its freshes. The great velocity which has been communicated to the river, cannot be supposed instantaneously to change upon passing the obstruction. Instead of that, we see a swift current shooting along in the line of the arches for a great way below the bridge, while powerful eddies run up in the line of the piers, casting up at length banks or shoals behind them, which tend, in their turn, to strengthen and prolong the original current and eddy. Whatever pains, therefore, we take to secure the pavement or inverted arch, this strong current must cut up and carry away the materials of the bed behind them; an operation which, if once begun, must constantly go on with increasing force. The water will have a fall over the lower end of the pavement, and will gradually wash out the foundation of the outer course of stones, which being immersed in water, will not be difficult to move. A few stones dropping out will add to the power of the stream, by roughening the bottom. Course will loosen after course, until the whole presents only a loose mass, ready to be torn up and swept away by the first ensuing flood in the river.

We could wish that what we have said here may induce persons properly qualified to turn their attention to the subject. We are convinced it is one of the most important departments of the art of bridge-building. Mathematicians have bestowed much time and pains on the equilibration of arches,—a matter about which the common bridge-builder seldom seems very solicitous. We have seen that, in reality, the usual speculations of that kind have hitherto led to no one useful practical result. Nay, if the deductions of the theory were to be followed too implicitly, they may lead, as in the case of the catenaria, and even the flat arch, to the proposing of weakness instead of strength, and craziness instead of stability.

But the security of the foundation is that about which the practical man is, with reason, most solicitous. He knows that it demands his greatest care. An error in that is irremediable, and there it is that his work generally fails. Give the ancient mechanician only a place to stand on, and he would have moved the world; give the modern engineer only a sure foundation, he will raise a structure as durable as the materials of which it is composed.

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Of the Sterlings or Extremities of the Piers.

Of the sterlings.

The reader must before this have seen, that there would be a great impropriety in forming the ends of our piers into planes at right angles to the stream; the water which is thereby shot off abruptly to each side, obstructs the general current by contracting the section, makes an increase of velocity necessary, which at the same time increases the action on the bottom, and hastens the downfall of the structure.

The bridge-builder, therefore, has in all ages endeavoured to obviate or diminish this contraction, by building projecting sterlings, or breakwaters, towards the stream, with the intention, as it were, of splitting the current, and conveying the waters more quietly under the arches. Those which point down the stream in rivers without reflux, were at first perhaps built only for the sake of uniformity; for although probably little less important than the other, they do not, as they are generally formed, seem calculated to serve any good purpose.

The form of the sterling has given rise to some discussion, and bridge builders do not yet seem agreed on what is the best. For the most part, they have been formed into an isosceles right angled triangle in the horizontal plan, having the right angle facing the stream; from a notion, perhaps pretty general among workmen, that this is of all angles the strongest. The projecting edge rises perpendicularly till above the surface of the water, and the spring of the arch; what is higher being merely matter of ornament, need not be mentioned here. At other times, the plans of these cutwaters or sterlings have been formed into two arches, of 60° each, described from the two angles of the pier, into a semicircle, or semi-ellipse, on the conjugate; or into other and probably fanciful figures, as in Fig. 7. Nor are these different methods without their advocates. Thus it is said for the right angle, that it divides the stream best, and a more acute angle would be too weak; that the semicircle and semiellipse, are best calculated to resist the shock of a loaded barge, or the like; and the Gothic intersecting arches, combine in some degree the advantages of both. But it is evident, we think, that if there be any form, which really deserves a preference over all others, it must be that which is adapted to the figure of the contracted stream; and which delivers the water in such a manner, as totally to fill the breadth of the archway. Unfortunately, however, our notions of the motions of fluids, are yet so far from being precise, that it is a matter of no small difficulty to discover what figure is best adapted to the purpose in view.

Plate LXXXI. Fig. 7.

That we may have the clearer conception of this matter, let us attend a little to the way in which a fluid in motion may be supposed to act upon any obstacle.

The particle moving in the direction EF (Fig. 8.) would strike the pier with the whole of its force, if the end of the pier was in the line AC, and the number of these particles will be as AD; but when the end is formed into the triangle ABC, the effect of

Plate LXXXI. Fig. 8.

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each particle on the plane AB is diminished in the proportion of the sine of its incidence EFB; and the action on the face being given, the effect of it in the direction BD, or parallel to the axis, will be found by still further diminishing it, in the ratio of the sine of obliquity. In the common case then, when the length of the pier is in the line of the stream, the resistance of the pier will be as the square of the sine of incidence, or it will be inversely as the square of the length of the face AB of the pier, that being a straight line. Otherwise, if EF represent the absolute force of any particle, draw the perpendiculars FG, EG, and GH, then FG exhibits the impulse perpendicular to the force AB, and FH the effect of that impulse in the direction of the axis BD; where, by the way, it may be observed, that if the angles DAB and DBA be equal, that is, if ABC be a right angle, then are FG and GE equal, also FH and HE; so that the absolute impulse on the sides of a rectangular wedge is just half the impulse on its base. We might pursue this mode of reasoning much further. We should find among other things, that the absolute impulse on right lined triangles, is less than on any curvilineal figure; that the impulse on cylinders, or the front of half cylinders, is just two thirds of the direct impulse on the base; that in all other curves, the nearer they approach to the right lined triangle, the less is the impulse upon them; and it is sufficiently evident, that the impulse will be always the less the more acute we make the vertex of that triangle, that is, the greater projection, and the sharper a point we give to the pier.

Plate LXXXI. Fig. 7.

This is the way in which De la Hire, Emerson, and other writers, have treated the subject. They give equations expressing the properties of different forms of sterlings; the whole being derived from the common theories of the resistance of fluids, conceiving the impulsion of the particles of the fluid to be the same as in the collision of hard bodies in free space.

But it is needless to follow this theory much further. We are convinced that it is founded altogether upon an improper assumption; and, at any rate, it is of small importance to the point in question. We think it may even reasonably be doubted, whether the best form of the sterling be merely that which gives least resistance to the stream. Should we not rather enquire after that which guides with most effect the water under the arches, and prevents the dangerous action on the foundation? At all events, we cannot concede, that the mode of action of the particles of water is the same which the above theory supposes, and which is commonly employed by writers on this subject. It would lead us into too wide a digression, to state the reasons which may be given against this doctrine of impulsion; besides, we shall have another opportunity of considering the subject more at length. See HYDRODYNAMICS.

We shall, therefore, at present, only exhibit, in a familiar way, the notion we have of the real action of a fluid in motion, which, although it may be more difficult to adapt to the precision of mathematical reasoning, will, we are convinced, be of more value to the practical builder, in giving him clear conceptions of the actions against which he is to provide. Let *ab* BA, Fig. 9. and *cd* DC, Fig. 10. be a thin film of water, which has advanced against the flat end AB, CD of the pier, and whether the first lamina of particles act by impulsion or not,—for this is not a place for metaphysical discussion,—let another and another succeed, until at length they constitute a plate or film *ab* BA, or *cd* DC, possessed of the properties of a fluid. This film will have had its direct motion destroyed by collision with the flat end of the pier. It cannot flow back, for it is stopped by the adjoining and following waters. Let us suppose it, for a moment, to be stationary. The second film now comes on, and being hindered from passing up to the pier by the first film, *ab* BA, can produce nothing like impulsion upon the pier, but it will propagate its force through the fluid film, in the way in which only a force is propagated through fluids; that is in every direction. The pier, therefore, will receive the impression of the second film unimpaired, but in the way of a pressure only, not as an impulsion.

Plate LXXXI. Fig. 9, 10.

In the mean time, the first film which is compressed between the pier, and the second film advancing, and the waters of which cannot as yet move off sidewise, being opposed by the other parts of the stream, hitherto supposed to be at the same level, must obey the hydrostatical law, and yield to the impression received, by its waters rising upwards, the only way in which they are free to move. There will be an accumulation C *ef* immediately before the pier. The second film will also be raised upon the same principle, but not so much; the third will be somewhat less than the second, and so on.

Now, it is evident, that the superficial waters of this accumulation being so much higher than the rest of the stream, must tend to slide off on all sides. This sliding off will cast them down in inclination towards the edges, and of course the greatest elevation will be just in the middle between A and B. At the same time, the same pressure which produces this accumulation, or, if it may be so considered, the very accumulation itself will propagate, in every direction, through that film, a corresponding pressure. This will enable the waters of the film to escape at each side, by pressing transversely on the passing current. No water could so escape without such an accumulation as we speak of; for the passing stream, being otherwise at the same level, would react with an equal pressure.

The notion, therefore, of the particles of water being reflected as in the collision of bodies in free space, cannot be entertained. Speculations founded upon that principle leave out the most remarkable feature of the case, viz. the fluidity of the water. Yet a trifling attention to that circumstance, renders the notions throughout the whole process much more familiar. We have established the fact of the accumulation of the fluid immediately in front of the pier. It is evident that no force whatever can be propagated through the fluid, without such an accumulation; we may therefore consider at once the accumulation as the cause, mark, and measure of every subsequent modification of the passing stream, and we will find it fully equal to the explanation of all the phenomena.

Of the film of water in the immediate neighbourhood of the pier, every part is urged laterally with the same force, viz. the excess of pressure produced

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by the accumulation. And, upon the supposition that all the water of the passing stream proceeds with the same velocity, which, by the way, is not strictly true, we might conclude that the deflection produced at the shoulder of the pier is the same at any depth. Now this deflection will have some proportion to the velocity of the stream. It evidently depends on the rapidity of the current. In the language of mathematicians, it will be a function of that velocity. Suppose, for a moment, that the velocity of the lateral discharge at the shoulder of the pier be equal to that of the current, it is plain that these waters, with those which are immediately contiguous in the stream, moving parallel to the side of the pier, would be projected in the diagonal of a square which had the side of the pier as its side. In that case we must suppose a certain space immediately behind the shoulder of the pier to be void of water; and at the same time, this sort of repulsion will produce a head of water, or accumulation in the stream, immediately beyond that void space. This accumulation will be propagated from the shoulder of the pier, as a centre across the arch, at the same time it will be carried down the stream; and accordingly, we do always see a wave, which proceeds from each sterling or abutment of an arch, and which meet perhaps a considerable way below the bridge: but, at the same time, and what is of more importance, the waters, which are, as it were, projected from the shoulder of the pier, are not at liberty to proceed in that direction; having on one side the void space above mentioned, they are repelled on the other, by the lateral and hydrostatic pressure of the general mass. This will, of course, act perpendicularly to their direction, and produce ultimately a sort of gyration or revolution. The superficial waters will likewise tend to run over and descend into the void; as they descend, they are exposed with the lower waters, to the lateral pressure of a greater depth; the gyration will become more rapid in descending; and its radius of curvature shorter. The void will assume the appearance of a hollow cone, the apex pointing downwards. The descending water will at length strike the bottom, and be reflected upwards; the motion of the apex will still be the most rapid, and will thereby produce that boiling appearance which we generally see towards the tail of the pier, and for a good way below the bridge. The general current is no sort of obstruction to this reflection from the bottom; it is perpendicular to its direction, and therefore neither helps nor hinders it, but it is the cause why the vortex is at first elongated in the direction of the stream, and why, after reflection from the bottom, it reappears considerably below the place of its original formation.

We have as yet taken into consideration the effect only of that film which lies in the immediate neighbourhood of the pier, and assuming it to pass laterally with a velocity equal to that of the general current; we have supposed that the primary deflection will be at an angle of 45°, but the second, third, &c. films, which are in the front of that, will have a similar lateral discharge, and will therefore have deflected a part of the waters of the general current, before they have reached the pier and the last film; but the deflection cannot be so great, because the accumulation is not so great. In fact, the first deflection is in a manner nothing. It is a nascent quantity; but each succeeding film having some room made for it by the deflection produced by the former, will be broader than it, measuring across the current, and will in its turn add a little to the former deflection; yet so, that the ultimate breadth can hardly be much greater than that of the pier, let the deflection be what it may.

All this while we are speaking of a square ended pier; and it is now clear, that the water which lies in a manner stagnated before it, is bounded on the plan by two curve lines, which have their convexity turned towards the axis of the pier, and are of course concave on the outside. If this, therefore, be the most advisable form of the sterlings of piers, it is, in all probability, different from any that have ever yet been constructed. Before, however, going further, we may observe, that the water in the front of the pier is by no means stagnant; passing in the direction of the stream, every film has a greater velocity than the succeeding; it has to supply not only its own waste by the lateral discharge, but that of all the succeeding, or rather, perhaps, its own waste, in passing through all the succeeding stages. At the commencement of the accumulation and deflection, the direction, as well as the velocity, is, in fact, that of the stream at the pier; the velocity in the direction of the stream vanishes, for the whole is deflected.

We conceive, therefore, although with great diffidence, that it is with impropriety, Newton has said, that the motions are the same as if a certain part of the water in front of the pier or obstruction were frozen, and he conceives this part to end in a point. No part of the water which is before the pier is perfectly stagnant, and it can therefore by no means be considered as frozen, neither can that portion of the water be supposed to come to a point; for, if we take, as the quantity of current intercepted by any pier, to the lateral discharge at the shoulder, so the breadth of the pier to a fourth proportional, it is plain that, roundly speaking, we have the distance on each side of the axis, to which this water extends, at that part of the stream where it may be supposed to be confounded with the general current; and this breadth must be something, since the discharge is something.

Giving up then the idea of making a sterling, or point to our pier, which shall be the same as the water that is supposed to stagnate before it, we think the best thing that can be done is to offer some maxims of construction, which, though they do not constitute a complete rule for the perfect formation of sterlings, will yet serve to improve the practice of the bridge builder, and preserve him from falling into gross and dangerous errors.

It is evident that all abrupt angles at the junction of the sterling with the pier are to be avoided: this part should be neatly and regularly rounded away, so as to prevent the gyration above alluded to, by giving the figure of the incipient part of the curve to the shoulder of the pier. Neither do we approve of the parallelism of the sides of the pier. A small convexity

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might be given them with much advantage, and without adding to the trouble of erection.

It were perhaps to be wished, that in the horizontal section of the curved part of the sterling, it should have a contrary flexure, so as to make the point of it, by being as sharp as possible, turn aside the waters with the greater ease; but this is attended with a manifest disadvantage. The point will then be liable to damage, and the most trivial variation in the thread of the stream, would produce in an increased degree all those gyrations we wanted to avoid; such points too, in navigable rivers, would be fatal to loaded craft. But while we do not attempt to give this acuteness to the pier in the horizontal section, we may acquire it with great facility in the longitudinal or vertical section. For that purpose, the lower courses should be made gradually to project before the upper, and if the formation of the pier admits of it, they may also be made to project on the sides; but care should be taken in so doing to attend to the rules we have formerly given for its stability, and not uselessly to increase the mass of the pier. As we descend, however, the courses of the sterling should become more acute, and project further up the stream, and thus the pier may be supposed ultimately to end in a mere point. In short, the figure of a pier, or sterling of this kind, will have a considerable resemblance to the sock of a plough,—an instrument which, in its application, has a considerable analogy with the sterling. It may be asked, whether these projecting courses should be left by the mason in steps, or trimmed away to a regular curve surface? We conceive, that there are good reasons for choosing to leave them in steps. For, independent of the saving of labour; when the current has any obliquity to the direction of the pier, it will flow over the acute point of the sterling, and being reflected by these steps, will be in a great measure prevented from injuring the bottom. The safety, however, of craft would seem to make it proper to round away the fore corners of the courses.

As to the point of the pier down the stream, its figure must, for the sake of uniformity, be assimilated to the other. It were well if we could in this case apply the contrary flexure above mentioned. The same difficulties are not to be dreaded; and although it may not be easy or agreeable to form it in the horizontal section, yet, by prolonging the tail down the stream further than usual, we will virtually obtain the same thing. The tail of the pier is of less consequence to the stability of a bridge, though it may certainly be contrived so as to facilitate the transmission of the water. Bridges seldom fail on that side; and as they are usually constructed, there is an eddy below the pier, which, although it obstructs the current, and injures the bottom, yet casts up a shoal below each pier that serves to protect the foundations of the building.

Bridges built in the way we have now mentioned, will appear greatly stronger than those of the usual construction; and they will really be so. Independent of their power of preventing the action on the foundation at the shoulder of the pier, which we have shewn in an early part of this section to be the principal cause of the decay of bridges, these sterlings will appear stout, curved buttresses, or as it were knees, to support the lofty side-walls; and by expanding the lower courses of the arch, advantage may be taken of the increased base, to cast the vault into a kind of groin, and thus give a greater degree of transverse strength to the whole structure.

This idea of extending the lower courses of the pier, and breaking, as it were, by degrees, the force of the current, is not new, and we do not give it as such. It may be seen with various degrees of perfection in many ancient and modern structures, particularly in the celebrated Pont St Esprit over the Rhone, which probably owes in a great measure its endurance to that expedient. But we wish merely to bring back to the builder, a principle which, in modern times, seems too often to have been lost sight of, and which was, no doubt, originally the result of a successful experience.

When necessity obliges us to form a bridge at a considerable obliquity to the current of a river, a danger is thereby incurred of producing a gyration on the *lee-bow* of the piers, which will be attended with all the evil consequences we have already explained. The obstruction to the current may be presumed to vary with the cosine of the angle of obliquity, and consequently the additional head must be as the versed sine of that angle. But if the sides of the river be parallel straight lines, the water-way under the bridge will just increase as the secant of the angle of obliquity, or inversely as the cosine. This will just counterbalance the obstruction caused by deflecting the current, leaving only the reaction of the bank, and the *weather-side* of each pier against the stream, to produce the deflection. There will be an accumulation on the one side of each arch therefore, and a depression on the other; while a strong gyration will take place on one of the shoulders of each pier, and which is likely to be attended with much danger. The water-way of the arch too is likely to be much contracted by this cause. All these appearances frequently occur, although the bridge appears to pass directly across the stream; because in such a case the bridge has been designed for the low waters, whereas the stream, when in flood, may have a very different direction. The course of the river therefore, above the bridge, should be carefully attended to, especially when the waters are high, and either corrected, or the position of the bridge adapted to it.

The effect of the gyration at the shoulder of the piers is curious; and, as it is one of the chief causes of their destruction, it is well deserving of attention. The beds of all rivers are porous, and will therefore be glutted with water; which again is everywhere pressed by the whole depth in the river: but at the shoulder of the pier there is a void, so that the bottom at that spot will not have the same vertical pressure that there is every where round it. Water will therefore rise out of the bottom at that place, like a spring or fountain, and so much the more forcibly as the void is deeper. Or, if prevented from issuing, it will exert a strong pressure upwards, upon whatever forms the bottom of the void. Now, this void being sometimes even four or five feet deep, we need

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not be in the least surprised, that not only gravel and shiver, but even large stones, are lifted in it, and shoved away by the vertical motion.

For these, and other reasons, it is difficult to ascertain the exact amount of the obstruction caused by obliquity. Like every other department of our inquiry connected with hydraulic principles, experiments are yet wanting to assist us in making this a subject of calculation.

Instead, therefore, of occupying the reader's time with a mathematical discussion, from which, at present, little benefit can be derived, we conceived it better to offer a few practical remarks on the methods by which the chief difficulties of such a case may be obviated.

With this view we will venture to recommend, that whatever the position of the bridge may be with respect to the stream, the lengthway of the piers and abutments should coincide with the direction of the current as nearly as may be. A considerable deviation is perfectly practicable. We have formerly shewn, that the position of the joints of the arch may be varied, perhaps 30° from that of equilibration, before any slide is thereby produced among the archstones: we surely have the same liberty here, and this even when the coursing joints are horizontal.

This advantage of friction may be even improved by dowelling or other means, if thought beneficial. But if the coursing-joints, instead of being made horizontal, be formed at right-angles to the side of the bridge, we have the very same advantages that would be obtained were the bridge perfectly direct.

Did the subject admit of it, we would here give the theory of those arches which are not straight on the ground-plan, as the arched top of a bow-window, and the like, some of which afford pretty examples of the application of the principles of equilibration, and all of which, by means of the gravity, friction, or cohesion of their parts, admit to a certain extent of being steadily and solidly constructed. But we shall, at another opportunity, take up this subject, and, in the meantime, we hasten to other matters more intimately connected with our present inquiries.

The centres or framings of carpentry, on which the arches are built, are, in many cases, objects of great anxiety to the builder: they form one of the most beautiful applications of the science of CARPENTRY. But as the principles upon which the more difficult kinds are designed will not be clearly understood, without entering into more detail than is consistent with our present subject, we shall only offer in this article some of the most approved designs, and refer the reader for their explanation and discussion of their comparative merits until we come to treat of CARPENTRY.

To the same place we shall also refer the subject of wooden and iron bridges, which intimately depend on the same science; their statical equilibrium admitting of the introduction of a principle essentially different from those employed in erections of stone.

The side-walls and wing-walls of bridges constitute a part not the least important of our subject. They have, especially the latter, to resist the pressure of a mass of earth behind them, forming the roadway up to the haunches of the bridge. Little has been written upon this subject in our language, and that little has been of no practical benefit. Muller and some others give us a few theorems respecting the thickness of the revetements in fortifications. Some of our elementary writers inquire a little into the same subject, as one case of the application of the doctrine of the centre of gravity. Their results, in almost every case, give a much greater thickness for walls of this kind, than is ever found necessary in practice. There can be no doubt that some principle must be overlooked in these investigations, which has a material effect in their application. We cannot say that the inquiries of Belidor, reckoned one of the first of scientific engineers, have been more successful. In another part of this work we shall offer a theory of retaining walls, in general founded upon principles that are perhaps new, and that has been confirmed by the results of experiment. This theory has been found to agree with the ideas of practical men. It is capable of application to all cases of the kind, and it comprehends them all. But in this place it would be embracing too large a field to bring it forward. (A. N.)

PART II. PRACTICE OF BRIDGE BUILDING.

Practice of Bridge Building.

WITH regard to the practice of bridge building, the chief objects may be arranged as follows:

SECT. I.

1. The situation.
2. The design.
3. The materials.

SECT. II.

1. The foundations.
2. The Piers and abutments.
3. The Centres.
4. The Arches.
5. The Spandrels and wings.
6. The Parapets.
7. The Roadway.

SECT. I.

On the Situation, Design, and Materials.

On the situation of bridges.

1. The situation is generally determined by local circumstances, in a town by streets, and in the country by the roads adjacent. If the bridge is of a great size, or the foundations difficult, it is frequently advisable to choose the most favourable situation for the bridge, even at the expence of changing the approaches. It is absolutely necessary that the access

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be commodious, but where practicable, from the approaches being in a curve, the general outlines of a bridge are seen to most advantage. The situation should be chosen where the river runs in a straight course for a considerable way above it, and where the channel rather contracts at some little distance below. The direction of the bridge should form a right angle with that of the river above it. Rock should, if possible, be obtained for the foundations of the abutments and piers; next to rock hard clay, or clay mixed with gravel, or otherwise firm gravel; but loose gravel, mud, or quicksands, are to be avoided as much as possible.

Designs for bridges.

2. When the situation has been determined, a map or plan should be made of the channel of the river and adjacent banks, also of the streets or roads which are to be connected with each end of the bridge. There should likewise be a section shewing the breadth of the bed of the river, and the form of the banks on each side. Upon this section should be marked the lines of high and low water, to be determined by the marks of the greatest floods, and the best information which can be procured from the oldest inhabitants, and most observing people in the neighbourhood. The consistence of the bed should also be ascertained, by boring with properly constructed augers, especially in the scite of the abutments, piers, and wing walls. There should likewise be a longitudinal section, shewing the declivity of the bed of the river, for at least 200 yards above and below the proposed situation.

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When these steps have been taken, and the nature of the materials ascertained, the engineer has obtained sufficient data to enable him to make a suitable design. We shall at present confine ourselves to bridges constructed with stone.

The leading objects in forming a design are,

1st, The passage for the water under the bridge.

2d, The making a perfect roadway over it: And,

3d, The decorations.

Passage for the water.

1. The number and dimensions of the arches must depend on the breadth of the river, the nature of the foundations, the height of the banks, and the quality of the materials to be used. If the foundations are good, the banks high, the stone hard, and to be procured of proper dimensions, large arches will be found advisable; if the reverse is the case, the size of the arches must be diminished, and their number increased; if more arches than one are required, their number should be odd, in order that there may be an arch in the middle of the river: the middle arch should be the largest, and those on each side should diminish, so that their springing remaining the same, their tops may form a declivity of about 1 in 24. When piers are placed in a river, the breadth between the abutments should be made greater than the natural breadth of the channel at that place, by about double the thickness of all the piers. Where the bed of the river is soft or loose, the increased width must be considerably more. In proportioning the thickness of the piers to the span of the arches, great latitude has been taken, as will appear from the following statement:

	Breadth of piers.	Span of arches.		Engineers.
Roman.	5	33	in the bridge at Rimini.	Unknown.
	11	25	.....do.....Vicenza.	
	38	110	.....do.....Alcantara.	
Middle ages.	32	115	.....do.....St Esprit.	Brothers of the Bridge.
	44	96	.....do.....Lyons.	
More modern times.	14	72	.....do.....Port Royal.	Mansard.
	19	106	.....do.....Orleans.	Hupeau.
	14	128	.....do.....Neuilly.	Perronet
	17	76	.....do.....Westminster.	Labelye.
	20	100	.....do.....Blackfriars.	Mylne.
At present.	18	77	.....do.....Perth.	Smeaton.
	14	90	.....do.....Dunkeld.	Telford.
	12	72	.....do.....Kelso.	Rennie.
	8	65	.....do.....Conon.	Telford.
	8	60	.....do.....Bewdley.	Telford.

From this statement it will be perceived, that the proportion has been varied from nearly one half to one ninth part of the span. As this important object has been discussed at length, when treating of the principles, it is only necessary here to observe, that the thickness of the piers, besides the span and shape of the arch, is affected by its rise, and in no inconsiderable degree by their own altitude, and the quality of the stone of which they are constructed; it is desirable, in order to lessen the obstruction to the waterway, to make the piers as thin as prudence will admit. The shape of the piers and arches should also be calculated, to form as perfect a water-way as possible; and on this account, all recesses and side projections, within reach of the water, should be avoided, and the connections of the abutments with the banks should be formed on the same principles.

Road-way.

2. If the bridge consists only of one small arch, and there is little intercourse, the road-way may be narrow and without footpaths; if the bridge is long, and there is much intercourse, the breadth must be increased, and have proper footpaths. Since wheel carriages have been in general use, few bridges, however small or remote from towns, have been made less than fifteen feet in breadth over the parapets; they are more generally made from 18 to 20 feet, in or

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near to considerable towns, from 26 to 30; and in or near large cities, they are made from 30 to 50 feet. The roadway should have a declivity from the middle of the length each way towards and beyond the abutments, of about 1 in 24; and the roadway, as well as the footpaths and parapets, should, near their extremities, diverge, to suit the approaches, whether one or more, at each end of the bridge.

Decorations.

3. The decorations should be varied, according to the situation and accompanyments. In the country, the utmost simplicity, consistent with distinguishing the essential parts, should be preserved; and even in the most splendid cities, or adjacent to palaces, all decorations should be kept perfectly subservient to, and in unison with, the essential parts: the neglect of this is a frequent error in designing bridges. Columns and entablatures, though proper in a Grecian temple, are ill suited to an edifice, where forms unknown to the Greeks are the leading features. As columns can only be placed over the piers and abutments, the entablature, intended to represent beams of timber, cannot be supposed to be wholly upheld by supports placed at such great distances from each other. And the introduction of columns, in place of carrying up the piers, deprives the superstructure of powerful buttresses in situations where they would prove very beneficial. The affectation of preserving the entablature upon a perfect level, has led to making the roadway along the bridge also level, which is nothing less than constructing, at a vast expense, a piece of road more imperfect than what is formed by the common labourer in the open country; and besides, this mode of construction gives an appearance of feebleness to the outlines of the bridge. This false taste was introduced by some of the French engineers, and has of late been, in some instances, copied in Britain. It cannot be too early reprobated, because bridges, when substantially constructed, remain for many ages, and are not easily altered.

In making out the design, the engineer should furnish a plan, shewing the form and dimensions of the foundations of the piers, abutments, wing walls, and wharf walls connected with the bridge; an elevation shewing the general facade; also vertical, longitudinal, and cross sections, shewing the construction of the interior parts. In large bridges, there should be a plan or horizontal section taken at about one third way up the rise of the arches, in order to shew the mode of filling up the spandrels. There should likewise be a plan of the roadway, footpaths, and parapets; besides separate drawings of all those parts which cannot be made sufficiently distinct in the general drawings. A specification should likewise be added, describing in words, the quality and dimensions of every part of the work, and the form and manner of its construction.

3. Materials.

Materials.

The materials consist of timber and iron for piles, cofferdams, caissons, scaffolding, and centres; of stone, lime, and sand for the masonry; also gravel for embanking at the ends, and forming the roadway over the bridge.

Of timber, oak is the most generally useful, and where exposed to be alternately wet and dry, is most durable; and next to it are pitch, pine, and fir grown from the seed. Under low water, elm is very suitable; it does not easily split, and is very durable: indeed, in this latter situation any timber is sufficiently durable. For gangways, scaffolding, and centers, sound fir timber of natural growth is the fittest; it should be free of knots and sap. In ties, bolts, and nails, malleable iron, made from wood charcoal, should be used. For receiving thrusts, forming supports, or dowelling stones, cast iron is the most proper.

With regard to stones for the piers, abutments, arch-stones, and parapets, no pains should be spared to procure the best which can be obtained at a reasonable expense. They should be of a quality not to be decomposed by the operations of the atmosphere, and they should not be intersected by cross seams. Where the quarry produces them with flat beds, much labour is saved; but when they are laminated, great care must be taken to have them laid on their natural beds. Their dimensions must, of course, be regulated by the magnitude of the work in which they are to be employed. If new quarries are to be opened, experienced workmen should carefully examine their quality; and specimens of the stone should be exposed for at least one winter and summer, in situations similar to that where they are to be used: experiments should also be made to prove their consistence. For the spandrels, wing walls, and backing, inferior stones may answer; but they should be such as can be laid with proper bond, and in regular courses. Those used on the external faces must be fit to withstand the effects of the weather: they should correspond with those of the piers and arches as to colour; and if regularly squared, should be brought from the same quarry, though they may be of smaller dimensions.

In procuring lime for mortar, it is of great importance that it will set or indurate under water; and where immediately exposed to agitated water, the outer edge of the joints should be laid with the British cement, discovered by Mr Parker, which in a few minutes becomes sufficiently hard. Where these cannot be procured, recourse must be had to substitutes: Burnt and pounded iron stone, scales from an iron forge, hard burnt tiles ground and mixed with quick lime, all become hard under water, and in damp situations. In all cases, clean, sharp, or angular sand, is a necessary ingredient; and in rubble work it is better for being very coarse, or approximating to clean small gravel. In making mortar, we have known great advantage derived from using water which contained a solution of iron, and was of a dark reddish colour, approaching to black, which is frequently found adjacent to peat mosses. The lime should be used when fresh slacked, and be well beaten, or made with a machine called a mortar mill.

In using sand, Vitruvius very judiciously makes a distinction between sea or river, and pit sand; of the former he allows two, and three of the latter, to one of lime in powder. Dr Higgins, who made many experiments, and published a treatise upon calcareous cements, recommends (by weight) one of lime to eight of sand. Loriot, in describing what he conceives the secret of the ancients, mixes a portion of unslacked lime ground to a powder, with the mortar

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which has previously been made up in heaps in the common way. In the specification for the Pont Royal des Thuilleries, by Mansard, and which seems to have served as a model in France, it is provided, that all the mortar of lime and cement, for the work under water, shall be composed of five parts, three of which shall be of good cement of tile, not bricks, and two of lime of Melun, well ground, or pounded together. For the common mortar, two of lime and three of sand. The Sieur Gabriel directs the same preparations for the bridge of Blois. Perronet, for the bridge of Neuilly, directs that the cement mortar shall be equal quantities of ground tiles from St Germain, or St Cloud, and lime; observing that this, on account of the quality of the lime, exceeds by $\frac{1}{10}$ the usual quantity employed in this sort of mortar. Of the other, or white mortar, he directs one third of slacked lime, and two thirds of sand. In the account of building the Eddystone light-house, Mr Smeaton gives a chapter upon water cements, from which we shall extract a few particulars; and earnestly recommend those who wish to be fully informed on this point, to peruse with attention the whole of that valuable chapter.

Mr Smeaton found, in many parts of England, limestone which produced lime, that when made into mortar grew hard under water. The best kind was at Aberthaw, in the Welsh side of the Bristol channel. He found the stone of Watchet, a small sea port in Somersetshire, had long been used in water works: it did not suit the purposes of agriculture. He traced the same sort of stone through the counties of Monmouth, Gloucester, Worcester, and Leicester, and thence by the vale of Belvoir into Nottingham, and Lincolnhire, at a place called Long Benington: he found it also in the counties of Dorset, Hants, Sussex, and Surrey. It has also long been procured in Lancashire, under the name of Sutten lime.

After mentioning the shape, appearance, and qualities of the sundry stones, he concludes, that as this sort of limestone is found, blue, grey, yellow, and white, also in thin strata and lumpish masses, sometimes very hard and sometimes comparatively soft; that its fitness does not depend upon those appearances and qualities, neither does it upon the matrix in which it is formed. But he found, that when burnt, all the water limes fell into a buff-coloured tinge, and all contained a considerable portion of clay.

He found once a reddish coarse deep brown sand stone, of a somewhat tender nature, which when burnt, pounded and sifted, and mixed with slacked lime, and made into a ball, became very hard.

He states, that limestone in general loses about $\frac{4}{5}$ of its weight by burning; that slacked lime produces double the measure of the burnt stone or shells; that when made into a paste, it occupies but half the space of the powder; that two measures of slacked lime made into a paste, and mixed with one measure of terras or puzzolano, makes about $1\frac{4}{10}$ of mortar.—The following Table is the result of his observations, experiments, and practice.

No.	Water lime with puzzolano.	Lime powder.	Puzzolano.	Common sand.	No. of cubic feet.
		Bushels.	Bushels.	Bushels.	
1	Eddystone mortar	2	2	—	2.32
2	Stone mortar	2	1	1	2.68
3	Do 2d sort	2	1	2	3.57
4	Face mortar	2	1	3	4.67
5	Do 2d sort	2	$0\frac{1}{2}$	3	4.17
6	Backing mortar	2	0	—	——
	Water lime with minion.		Minion.		
7	Face mortar	2	2	1	3.22
8	Do Calder composition	2	1	2	3.57
9	Backing mortar	2	$0\frac{1}{2}$	3	4.17
10	Do 2d sort	2	$0\frac{1}{4}$	3	4.04
	Common lime with terras.		Terras.		
11	Terras mortar	2	1	—	1.67
12	increased	2	2	1	2.50
13	further	2	1	2	3.45
14	still further	2	1	3	4.35
15	Terras backing mortar	2	$0\frac{1}{2}$	3	3.50
16	2d sort	2	$0\frac{1}{4}$	3	3.37
	Common lime with minion.		Minion.		
17	Ordinary face mortar	2	2	2	2.75
18	2d sort	2	1	3	4.34
19	Ordinary backing mortar	2	$0\frac{1}{2}$	3	4.05
20	2d sort	2	$0\frac{1}{4}$	3	3.92

Practice.

He observes upon this Table, that these materials are all supposed in the dry state: That the lime and puzzolano must be thrown into the measure or vessel with the same degree of force.

He denominates minion to be what falls from the outside of the lumps of iron stone.

He allows the day's work of a man for beating every bushel of terras; that is, two bushels of lime powder and one bushel of terras. The bushel is the Winchester level with the border. He allows iron forge scales, when sifted clean from dirt and clay, as equal to as much terras or puzzolano.

His mode of working is "to mix the due proportion of the lime and the puzzolano, the terras or the minion, together in dry powder; and it will also be well to have at least one third of the sand (either fine or coarse), likewise dry; put as much water to the lime as, with a shovel or beater, you can bring it to a paste of a moderate consistence, but rather more wet than to be properly used as mortar in that state; then by degrees beat in the moist sand, and afterwards the dry, bringing it to a consistence by beating after every addition. The dry sand is intended to dry up the superfluous moisture, so as to render the mortar fit for immediate use; and if this has not brought it to a sufficient stiffness, let it lie till it is inclined to set, and then beat it up to a due consistence; or if immediately wanted, beat in a little dry lime powder, always however faithfully remembering not to terminate beating till the mass has got all the toughness that you find it will acquire by beating."

In Scotland, most of the limes harden under water: that from Lord Elgin's great lime works on the river Forth, from Portsoy on the Murray coast, and from the island of Lismore, in the Linnhe loch on the west coast: the last is the best, it slacks into a buff colour. Mr Telford discovered its qualities when searching for materials for the western district of the Caledonian canal, by observing in a park wall, belonging to the Marquis of Tweeddale, built against spongy ground, and where water oozed through, that the mortar was hardest, and threw out stalactites; also that it was quite perfect on the top of the wall, where worked among small stones, without coping of any sort.

Sect. II.

On the Foundations, Piers, Abutments, Centres, Arches, Spandrels, Parapets, and Roadway.

Foundations.

The situation, general design, and materials having been determined, the next step is to prepare the foundations; and if the water is shallow and upon rock, or other matter sufficiently firm, the operations are very simple; nothing being necessary but to turn the water, by means of a mound of clay, from the space to be occupied by the abutments and piers successively, to clear and level the ground, and to proceed with the masonry: But when the water is deep, and the foundation soft mud or sand, or loose gravel, the difficulties frequently require all the art of the most experienced engineers. Even when the foundation is clay or rock, if the water is deep, and the currents from tides or land floods considerable, few operations require more talents or attention.

Practice.

Various are the schemes which have been resorted to, in order to avoid or lessen the difficulties attending the laying the foundations of bridges. 1. A situation has been chosen in the bend of the river, to which, after the bridge was constructed, a new river course has been cut. 2. A side channel has been formed, by which the whole, or greatest part of the water, has been drawn off during the time the foundations were put in the old river course; and the water, so drawn off, was afterwards returned to its original channel. 3. Mill weirs on the river, below the proposed situation, have had their sluices drawn, or have been partly removed for a time. 4. And when there happened to be no weirs, the bed of the river has been deepened, for a considerable distance, in order to lower the water at the intended scite.

When the depth of the water has been reduced as much as possible, it remains to be determined in what manner to proceed. Anciently, (as in the case of London bridge,) in deep rivers, the foundations of the piers were made by merely driving piles all over the space, so that their heads stood level with low water; the spaces between them were filled with loose stones, and the masonry begun upon the top of them; but the piers were immense masses, and required to be protected by sterlings; which, leaving a very confined water-way, created a head and velocity which tore away the bed of the river immediately below the piers. This mode having been long disused, we shall proceed to consider,

1st, Batterdeaux, or Cofferdams; and,

2dly, Caissons.

Both methods have been employed with success: The greatest of the modern bridges in France, that is to say, Orleans and Neuilly, were constructed by means of the former; whereas, in England, the bridges at Westminster and Blackfriars were accomplished by the latter mode.

Of Cofferdams.

Of Cofferdams.

We shall first describe the measures pursued both in France and Britain, in working by means of batterdeaux or cofferdams.

The forms of these are varied according to local circumstances, and the ingenuity of the persons who have the direction of the work.

1. By driving two rows of vertical piles and plank piles, and filling the space between them with clay.

2. By driving main piles, and working with strong planking, laid in a horizontal position.

3. By driving one row of guaging piles, and filling the spaces between them with pile planks driven vertically.

The most eminent French bridge engineers, viz. Mansard, Sieur Gabriel, Gautier, Hupeau and Perronet, in constructing their greatest bridges with cofferdams, directed piles, from nine to ten inches diameter, to be placed from three to four feet from centre to centre, and driven from three to six feet into the bed of the river, (if composed of mud; clay, or gravel,) and to rise (in their rivers) six feet above low water mark. Their pile planks were nine to twelve inches broad, and four inches thick; one frame contained 16 of these pile planks, which were